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by

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Do Mean Reverting based trading strategies outperform Buy and Hold ? *

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Abstract

If prices of assets are not aligned to their net present value, a trading strategy may be implemented when actual prices revert to fundamentals. This hypothesis is informally tested in real-time using a trading strategy which consists of identifying whether the equity index is over or under-priced. The fundamental price is constructed in real time using the net present value approach which requires the series for expected dividends, expected returns and expected dividend growth rate. These series, typically unobservable, are derived from a structural state space model and econometric models. The performance of the rules depend on the frequency of the data. Cyclical behaviour within the the one year frequency may not be discarded. The trading strategy performs poorly implying that mean reversion may not be uncovered in real-time. However a hybrid variant of the structural and econometric model is shown to outperform the passive Buy and Hold strategy

Key Words: Trading Rule,Asset Pricing, State Space Modeling
JEL References: G12, G14, G17

*Preliminary and incomplete. Please do not quote without consulting the author first.

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1 Introduction

If an asset is overpriced or underpriced relative to its true valuation, mean reversion towards its true value may be anticipated in subsequent periods. In asset equilibrium search models, rational agents will arbitrage this riskless gain so that the price is aligned to the fundamental value of the asset. In the context of an equity market, if it is possible to identify whether the market is underpriced or overpriced relative to the fundamental value, a simple strategy would be to go long on equity or bonds (depending on the direction of the mispricing). The trading strategy is developed in line with time varying expected returns and expected dividend growth. Previous literature showed that the stock market is too volatile to be justified by the rational expectations present value model (**Shiller and Beltratti (1993), Poterba and Summers (1988)**). The rule I develop attempts to exploit the volatile nature of stock markets, while resting on the assumption of Efficient Markets (as defined in **Timmermann and Granger (2004)**) and rational expectations.

An issue of interest in the real-time framework is the computation of the fundamental value of the asset. Uncertainty exists in both the model that drives the value of the asset. Morevover,the variables in the model are latent. I construct four series of theoretical price assuming one state space structural model, two models of dividend forecasting and a hybrid model reconciling the state space model with forecasts.price. At this stage, rolling and recursive window forecasts of dividends, price, returns and dividend growth rate are the best real time approximations, given that agents expectations of these variables are latent.The theoretical prices are then compared to the actual price to identify the asset that should be held.

The structural model that is used to derive the expected returns and expected dividend growth is based on the present value identity of **Campbell and Shiller(1988).** The model invokes a state space structure where one of the measurement equations is the present value identity. The model is formalized in **Van Binsbergen and Koijen (2010).** The intuition of the model is that expected returns and expected dividend growth are unobservable to the econometrician in real time. However we do observe the realized values. The Expected Returns and Expected Dividend Growth series are filtered from realized observations based on the Kalman procedure, where expectations are updated as a new observation of the realized value. The law of motion for the price dividend ratio from the Campbell and Shiller(1988) identity is derived, assuming that the latent variables (expected returns and expected dividend growth) follow an autoregressive process. The Kalman Filter is applied to the model and the parameters are optimized using the conditional Maximum Likelihood procedure.

Consistent with rational expectations and the Efficient Markets Hypothesis, the structural decomposition of the expected returns and expected dividend growth rate rely on the Price -Dividend Ratio as being the key variable governing the law of motion and having theoretical links with the two components we want to derive. As such, the build of the state space model involves the Price Dividend Ratio and realized dividend growths as being the measurement equation with the expected returns and expected dividend growth processes forming the state equation. The interesting aspect of such a model, just as in the recursive and rolling model, is that it is robust to structural breaks and does not require the estimation of a large number of parameters **(Rytchkov 2007)**.

A more subtle and realistic approach to deriving the real-time expected values in the present value formulae is by simply forecasting the variables out of sample. The data generating process being unknown, simple econometric models are put forward. For instance, both expected returns and expected dividend growth rate are proxied by forecasts of a constant mean model. Price is forecast from a growth model with a constant mean and a trend term. Dividend forecasts are proxied by an autoregressive process and a pro-cyclical business cycle factor. To allow for expanding information sets, the recursive window models was used.

Having derived the expected returns and expected dividend growth series, the rational present value (fundamental price) of the index is computed using two models. The fundamental price is compared to the realised price to decide whether to go long on equity or bonds. If the theoretical price is higher than the actual price, implying the market is underpriced, the proper strategy would be to go long on equity, assuming that actual price revert to the fundamental price. On the other hand if the market price is high relative to the theoretical price, this will lead to a capital loss and hence the proper strategy would be to shift the assets from equity to bonds.

2 Literature review

In this section we look at related studies on the trading rule, the net present value model as applied in the financial markets, and the state space models application in the derivation of the expected returns and expected dividend growth.

2.1 Trading Rule

The theoretical underpinning of the rule involves the comparison of the Actual Price with the fundamental price (present value) in order to define whether it is profitable to go long or short on it. Theoretically, the fundamental price is determined by the expectations of agents and the type of process they used to model the data generating process of dividends **(Timmermann 1993)**. Any difference between the fundamental and actual price, will lead agents to revise their expectations and their own behavior will move the market back to equilibrium. Hence a profitable opportunity might arise during the adjustment of the market price towards the fundamental value. For instance if the stock market price is higher than the one postulated by the rational valuation, implying that the market is overpriced, reversion to fundamentals implies that price will move downwards, hence leading to a capital loss. On the other hand if the stock market price is underpriced relative to the market, we should expect that the price will rise then reaping a capital gain.

The model was first put to use in **Bulkley and Tonks (1989)** where the expost rational price was compared with the net present value. They successfully showed how marginal variation in the estimated dividend model may cause excess volatility. They also showed that the rule works for the UK market. Timmermann(1993) shows a similar result. **Rambaccussing (2009)** tests the rule in a real time context by assuming that agents have an econometric model from which they can forecast dividends.The expected returns and dividend growth rate are generated in the same way. He finds that the rule works better for longer holding horizons.

2.2 Net Present Value

The Net Present Value approach to deriving the theoretical price involves the assumption of a general equilibrium economy where any riskless return may be arbitraged away. An interesting derivation of the model may be found in Cochrane (2002). The basic premise of the net present value is that the theoretical price is derived by the infinitely discounted payoffs from the asset. The theoretical price (or NPV) is given by :

$$P_{1t}^{*} = E_t \sum_{i=1}^{\infty} [r_{t,t+j} D_{t+j}] \tag{1}$$

where E_t is the expectations operator at time t, $r_{t,t+j}$ is the return from time t to t+j and D_{t+j} relates to the dividend at time t+j.

If dividends are growing at a rate g_{t+1} and $E_t\,[r_{t+1}]$ is greater than g_{t+1}, equation **15** can be written as:

$$P_{2t}^{*} = \frac{1}{E_t[r_{t+1} - g_{t+1}]} E_t[D_{t+1}] \tag{2}$$

Equation 17 is used to compute the REPV price. The rational valuation formulae has been extensively to derive the Efficient Markets Hypothesis price.For applications, see **(Cuthbertson & Hyde, 2002)**, **(Kanas, 2005)**, **(Rangvid, 2006)**, **(Shiller & Beltratti, 1993)**, **(Allen & Yang, 2004)**, **(Caporale & Gil-Alana, 2004), (Strauss & Yigit, 2001), (Bohl & Siklos, 2004)** and **(Mills, 1993)**

An alternative specification more suited for time varying returns and long horizon uncertainty may be :

$$P_{t}^{*} = \frac{1}{1 + E_t[r_{t+1}]} E_t[D_{t+1}] + E_t[P_{t+1}] \tag{3}$$

This formulation takes into account that prices may be driven by two growth components, namely growth in prices and growth in dividends.

2.3 Time Varying Expected returns and Dividend Growth

The asset pricing model applies state space modeling to the derived present value of Price Dividend ratio. The objective of applying the state space model is to derive expected returns and expected dividend growth which are both latent variables but which are linked through the price dividend ratio identity. Decomposing the price dividend ratio between expected growth and returns is in line with the net present value approach and is formed with rational expectations foundations. According to theory, if prices and dividends are cointegrated, then all the variation in the dividend -price ratio must come from the variation of expected returns and dividend growth. Past empiricism has found the latter to be unpredictable, hence in this sense, all the variation in the price dividend ratio comes from the changes in expected returns.

The methods for deriving the expected returns and expected dividend growth rate can include various classifications such as the simple trend, predictive Ordinary Least Squares,. Bayesian models and State Space models. The present study uses state space modeling with specific reference to the Kalman Filter. Since both expected returns and expected dividend growth rate are unobservable, the state space model can be used to provide most efficient estimates of these two variables given observed data. The filtering technique used to uncover expected returns has been used widely in the literature. **Conrad and Kaul (1988)** apply the Kalman Filter to extract expected returns from the history of realized returns. The objective was to attempt to characterize the random nature of expected returns and test whether the latter was constant.**Brandt and Kang (2004)** investigated the relationship between expected returns and volatility. The model conditional mean and volatility of returns as unobservable variables which follow a latent VAR model and filter them from observed returns. In the same line of thinking, **Cochrane (2008)** shows that the VAR model can be represented in state space form. **Pastor and Stambaugh (2009)** use the Kalman Filter to analyze the correlation between predictors and expected return in the forecast of returns. Since imperfect exogenous predictors imply that there are no errors in variables bias, the kalman filter to uncover the unobservable expected returns from realized returns. **Koijen and Van Binsbergen (2010)** use the state space model to model cash invested and market invested dividends

3 Methodology

3.1 Net Present Value Model

In this section, we derive the net present value relationship between the Price Dividend Ratio and expected returns and expected dividend growth. Interestingly the series is developed from a theoretical assumption that both expected returns and dividend growth rate follows an autoregressive process of order 1. We start by defining some standard equations in the literature and then we derive the Campbell and Shiller (1988) log linearized model:

The rate of return is defined as

$$r_t = \log(\frac{P_{t+1} + D_{t+1}}{P_t}) \tag{4}$$

The Price Dividend ratio is defined as

$$PD_t = \frac{P_t}{D_t} \tag{5}$$

The Dividend Growth rate is defined as

$$\Delta d_{t+1} = \log(\frac{D_{t+1}}{D_t}) \tag{6}$$

One of the important assumptions that we put forward for the process of expected returns and dividend growth concerns the order of the process. The intuitive idea concerning the functional form of the process is that there should be in theory near to the data generating process. However, this endeavour of finding a best model is hectic and involves a lot of data mining. We shall assume our own functional form of the model. The mean adjusted conditional expected returns and dividend growth rate are modelled as an autoregressive process as in equations 7 and 8 respectively :

$$\mu_{t+1} - \delta_0 = \delta_1(\mu_t - \delta_0) + \varepsilon^{\mu}_{t+1} \tag{7}$$

$$g_{t+1} - \gamma_0 = \gamma_1(g_t - \gamma_0) + \varepsilon^{g}_{t+1} \tag{8}$$

where $\mu_t = E_t(r_{t+1})$ and $g_t = E_t(g_{t+1})$

Equation 7 and 8 relates to the mean deviation of the expected returns and expected dividend growth rate where δ_0 and γ_0 represents the unconditional mean of the expected returns and dividend growth respectively. δ_1 and γ_1 represents the autoregressive parameters. ε^{μ}_{t+1} and ε^{g}_{t+1} represents the shocks to the expected returns and the dividend growth rate processes. $\varepsilon^{\mu}_{t+1} \sim N(0, \sigma^2_{\mu})$ and $\varepsilon^{g}_{t+1} \sim N(0, \sigma^2_{g})$. However, we do not implement any restrictions between the covariance of $\varepsilon^{\mu}_{t+1}$and ε^{g}_{t+1}because a shock to the expected return process might actually affect the dividend growth process as well.

The realized dividend growth rate are defined as the expected dividend growth rate and expected returns and the unobserved shock ε^{d}_{t+1}, where by :

$$\Delta d_{t+1} = g_t + \varepsilon^{d}_{t+1} \tag{9}$$

ε^{d}_{t+1} and g_t are assumed to be orthogonal to each other. $\mathrm{E}(\varepsilon^{d}_{t+1}, g_t) = 0$.

The Campbell and Shiller (1988) log linearized return equation (derived in appendix 1) may be written as :

$$r_{t+1} = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - pd_t \tag{10}$$

where $pd_t = E[\log(PD_t)], \kappa$ is an arbitrary constant defined as $log(1 + exp(\overline{pd})) - \rho\overline{pd}$ and $\rho = \frac{\exp(\overline{pd})}{1+\exp(\overline{pd})}$

The equation can be further be reduced to :

$$r_{t+1} = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - pd_t \tag{11}$$

To study the dynamics of the price dividend ratio, the process may be written with pd_t being the subject of the formula:

$$pd_t = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - r_{t+1}$$

The full derivation of the of the price dividend ratio model is explained in appendix 1.

By replacing lagged iterated values of pd_{t+1}in the equation, the process may be written as :

$$pd_t = \sum_{i=0}^{\infty} \rho^i \kappa + \rho^{\infty} pd_{\infty} + \sum_{i=1}^{\infty} \rho^{i-1} (\Delta d_{t+i} - r_{t+i})$$

$$pd_t = \frac{\kappa}{1-\rho} + \rho^{\infty} pd_{\infty} + \sum_{i=1}^{\infty} \rho^{i-1} (\Delta d_{t+i} - r_{t+i})$$

3.2 State Space Model

The state space model makes use of a transition equation and a measurement equation. The Kalman Filter best illustrates the dynamics of the estimates of $\mu_t$and g_t.The model parameters are estimated before making the forecasts. The maximum likelihood estimator is used to obtain the parameters of the Kalman filter. The Maximum likelihood is optimized using the MaxBFGS procedure.

There are two transition equations, one governing the dividend growth rate and the other one governing the mean return:

$$\widehat{g}_{t+1} = \gamma_1 \widehat{g}_t + \varepsilon_{t+1}^g \tag{12}$$

$$\widehat{\mu}_{t+1} = \delta_1 \widehat{\mu}_t + \varepsilon_{t+1}^{\mu} \tag{13}$$

the two measurement equations are given by :

$$\Delta d_{t+1} = \gamma_0 + \widehat{g}_t + \varepsilon_{t+1}^d \tag{14}$$

$$pd_t = A - B\widehat{\mu}_t + B\widehat{g}_t \tag{15}$$

Equation 13 can be rearranged into 15 such that there are only two measurement equations and only one state space model.

$$\widehat{g}_{t+1} = \gamma_1 \widehat{g}_t + \varepsilon_{t+1}^g \tag{16}$$

$$\Delta d_{t+1} = \gamma_0 + \widehat{g}_t + \varepsilon_{t+1}^d \tag{17}$$

$$pd_{t+1} = (1-\delta_1)A + B_2(\gamma_1 - \delta_1)\widehat{g}_t + \delta_1 pd_t - B_1\varepsilon^{\mu}_{t+1} + B_2\varepsilon^{g}_{t+1} \quad (18)$$

Equation 16 defines the transition (state) equation. The measurement equation relates the obsevable variable to theunobserved variables. In our case this is given by equation 17 and 18. A is equal to $\frac{\kappa}{1-\rho} + \frac{\gamma_0-\delta_0}{1-\rho}$, $B_1 = \frac{1}{1-\rho\delta_1}$, $B_2 = \frac{1}{1-\rho\gamma_1}$.

Equation 17 and 18 relate to the measurement equation. This can be put into a state space form as shown in appendix. Since all the equations are linear, we can implement the Kalman Filter and obtain the likelihood which is maximized over the following vector of parameters.

$$\Theta = (\gamma_0, \delta_0, \gamma_1, \delta_1, \sigma_g, \sigma_\mu, \sigma_d, \rho_{g\mu}, \rho_{gD}, \rho_{\mu D})$$

The individual elements of the state and measurement vectors are given in appendix 2.

The filtered series for the expected dividend growth is just taken to be the first element for the state vector X_t (Refer to appendix 2 for a more detailed explanation). The state vector is derived according to the following update :

$$X_{t|t-1} = FX_{t-1|t-1} \quad (19)$$

In the case of the demeaned expected returns, the expected returns is defined as :

$$\widehat{\mu}_{t-1|t-1} = B_1^{-1}(pd_t - A - B_2\widehat{g}_{t-1|t-1}) \quad (20)$$

3.3 Expected Future Dividends

In this section, we look at the performance of a trading rule using the expected returns and expected dividend growth rate we derived in the earlier section. The theoretical price or net present value of the market equity is derived by using the net present value approach to returns. This is given by equation 21

$$P^*_{1t} = \frac{1}{\mu_t - g_t} E_t[D_{t+1}] \tag{21}$$

where $\mathrm{E}_t[r_{t+1}]$ and $E_t[g_{t+1}]$ are the expected returns and growth series that we derive using the state space model.

$E_t[D_{t+1}]$ is a real time valuation of expected dividends, which is computed as :

$$E_t[D_{t+1}] = D_t(1 + g_t)$$

The expected future dividend based on expectations at time t is made up of the realized dividend compounded with the expected growth rate. It should be noted that at time t, the realized value of the dividends is known.

3.4 Econometric Models

In this section we detail the econometric models to forecast the various variables that may be used to compute the net present value. The models for each variable are namely:

Dividends:

$$d_{t+1} = \beta_0 + \beta_1 d_t + u_t \tag{22}$$

$$d_{t+1} = \beta_0 + \beta_1 \frac{e_t}{p_t} + u_t \tag{23}$$

where d_{t+1} is the forecast dividend in natural logarithm. Equation 22 states that the forecast dividends are forecast from an autoregressive process. Equation 23 forecasts the dividend from the earnings price ratio. The earnings price ratio takes into account business cycle variation.

Price forecasts:

$$p_t = \beta_0 + \beta_1 t \tag{24}$$

The growth in price is forecast from a simple model with deterministic trend.

Expected returns and Dividend growth

$$g_{t+1} = \beta_0 + u_t \tag{25}$$

$$\mu_{t+1} = \beta_0 + u_t \tag{26}$$

The expected returns and dividend growth is forecast from a constant mean model.

The variables are forecast out of sample. from the estimated parameters. The model of estimation is the recursive window which allows the information set to expand with new realized values.

3.5 Trading Rule

Five trading strategies are considered. The general trading rule can be summarized as follows:

1) Go long on the equity index if

$$\frac{P_t^*}{P_t} > 1$$

2) Go long on the risk free asset if :

$$\frac{P_t^*}{P_t} < 1$$

where P_t^* is the theoretical price or the net present at time t.

The first model (Model 1) involves computing 2 with the values that are derived from the state space model. Model 2 looks at the contrarian option using the present value computed for model 1. In other words, it predicts going long on the equity market if the market is overpriced and going long on bond markets if the equity market is underpriced. Model 3 is a hybrid computation of 3. The discounted dividend part is computed using the state space model, while the price is computed using the forecast model 24. Model 4 involves forecasting dividends from the autoregressive process (equation 24) and discounting it with the forecast dividend growth rates (equation 25) and expected returns (equation 26). Model 5 involves the same variables as model four except that dividends are forecast from model 22.

4 Data and Results

The annual series of bond returns, realized market returns, dividend, earnings price is retrieved from Shiller's website for the period 1870 to 2008. The indicators and returns variables are controlled for inflationary tendencies. The state space model is optimized using data from December 1899 to December 2008. For the regression models, the sample 1870 till 1899 is used as the inital estimation sample.

4.1 Results

In this section the results on the state space model are reported along with the properties of the series. The results of the optimization is reported in table 1.

Parameter	Coefficient	Std error
$\boldsymbol{\gamma}_0$	0.012	0.0127
$\boldsymbol{\delta}_0$	0.045	0.0176
$\boldsymbol{\gamma}_1$	0.166	0.0940
$\boldsymbol{\delta}_1$	0.945	0.0456
$\boldsymbol{\sigma}_g$	0.07	0.0111
$\boldsymbol{\sigma}_d$	0.014	0.0080
$\boldsymbol{\sigma}_\mu$	0.088	0.0315
$\boldsymbol{\rho}_{g\mu}$	0.445	0.0894
$\boldsymbol{\rho}_{\mu d}$	-0.240	0.0021

Table 1:
Optimization of State Space Model.The optimal values model are reported in the second column. The optimisation procedure was MaxBFGS. The associated standard errors are computed analytically from the Hessian Matrix.

The results from table 1 show that the unconditional mean returns turns out to be 4.2 % where as the annual dividend growth is less namely 1.2 %. The standard errors indicate that there is a high variation around the intercept term of both state equations. Interestingly,the coefficient from the autoregressive parameter of expected dividend growth is low implying that expected dividend growth rate in itself is not cannot be predicted from past lags, further reinforcing the empirical evidence that dividend growth is unpredictable. However, the interesting result comes from the autoregressive coefficient of the expected returns which tends to depict high persistence. It is worth mentioning that this contrasts the findings on the actual market return which does not exhibit any persistence in the first moment. The positive correlation between expected dividend growth and expected returns illustrates that a shock to the net present value is partially offset by the model.

4.1.1 Summary Statistics

The Summary statistics of the expected returns and dividend growth rate are shown in table 2 :

	$\mathbf{R}_t$	$\boldsymbol{\mu}_t$	$\boldsymbol{\Delta}\mathbf{d}_t$	$\mathbf{g}_t$
Mean	0.057	0.059	0.015	0.051
Std. Deviation	0.185	0.038	0.115	0.011
Skewness	-0.612	-1.061	-0.716	-1.012
Kurtosis	3.341	3.879	7.812	7.357
Jarque-Bera	7.33	23.9	114	104

Table 2:
Summary Statistics. The mean, standard deviation,and other moments are reported for the realized and expected values for returns and dividend growth.

Both $\boldsymbol{\mu}_t$ and $\mathbf{g}_t$ have unconditional mean properties close to the realized values. The expected variables possess a lower volatility than the observed values. The third moments illustrate a higher negative skewness value, illustrating that the variance in expected returns are explained by extreme deviations from the mean. Further tests on stationarity are provided in table A.3 of the appendix. The null hypothesis of stationarity is not rejected in most cases. However we find that the stationarity results for the expected returns in terms of the Robinson- Lobato (p value = 0.418) test tend to be weak. The non stationarity test results show that there is high evidence for stationarity in the actual returns, actual and expected dividend growth. However the expected returns is purely non stationary according to the various tests. Although the test show that there are non stationary models involved, we make out that it is a near unit root process as show by the autoregressive coefficient in table 1.The results point out that expected returns may follow a random walk.

4.2 Time Variation

In this section, we present the results of a test of constant expected returns and expected dividend growth rate. We perform a bootstrapped test for large sample purposes. Constant expected returns and constant expected dividend growth rate implies that the autoregressive parameters are set to zero. A stronger assumption is that the standard error is also equal to zero. The null hypotheses can be written as follow

For expected returns,

$$H_o : \delta_1 = \sigma_\mu = 0$$

For expected dividend growth rate,

$$H_0 : \gamma_1 = \sigma_d = 0$$

The likelihood ratio test is given by:

$$LR_\mu = \overline{L(\Theta)} - \overline{L(\Theta_1)}$$

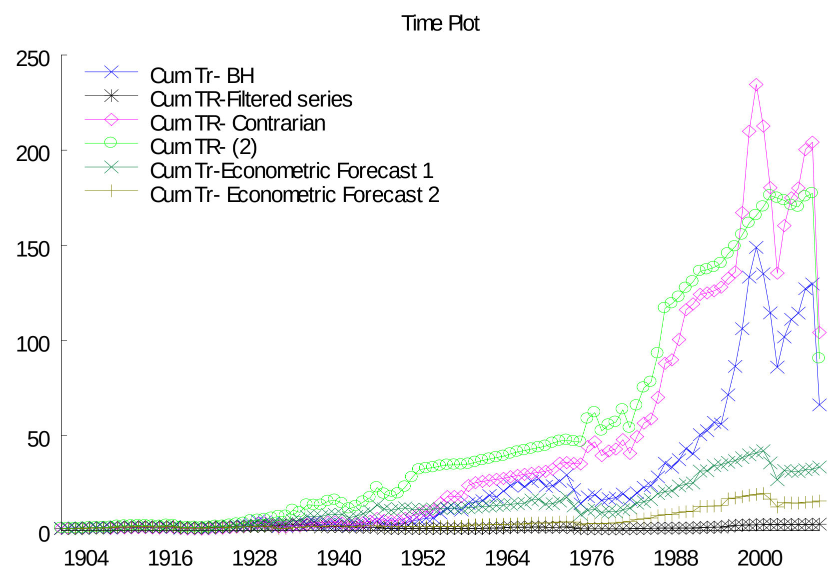
$$LR_d = \overline{L(\Theta)} - \overline{L(\Theta_2)}$$

where $\Theta_1 = (\gamma_0, \delta_0, \gamma_1, \sigma_g, \sigma_d, \rho_{g\mu}, \rho_{gd}, \rho_{\mu d})$
$\Theta_2 = (\gamma_0, \delta_0, \delta_1, \sigma_\mu, \sigma_d, \rho_{g\mu}, \rho_{gd}, \rho_{\mu d})$

In the above, $\overline{L(\Theta)}$ denotes the average likelihood for the model estimated with Θ. Under each of the null, the likelihood ratio follows a chi-square with 2 degrees of freedom. The number of bootstrapped replication was 300. $LR_\mu$and LR_d was 1824.6 and 1820.2 respectively, hereby leading to the conclusion of time varying expected returns and expected dividend growth rate.

4.2.1 Trading Rule

In this section we compare the results of the initial trading rule as the one stipulated in the model with monthly data. We also compare the trading rule where the expected returns, dividend growth rate are estimated from simple econometric models. The rule was unsuccessful for most of times. However a hybrid structural model shows that the Buy and Hold model may be bested. The hybrid model computes the net present value of the form 3. This involves using the computed parameters from the state space equation to derive $E_t[r_{t+1}]$ and $E_t[D_{t+1}]$. The $E_t[P_{t+1}]$ is estimated from two functional form models presented in the appendix. We report firstly the the cummulated wealth under the following strategies: Passive Buy and Hold, the net present value approach (2), a strategy contrarian to the latter and the hybrid model(3). Figure shows the plot of the four strategies in terms of accumulated wealth, if the rule is applied each and every period.



Plot of Trading Strategies. The following plot shows the performance of the different trading strategies over time.

	BH	Ideal Ret	M 1	M2	M 3	M 4	M5
Mean	0.038	0.107	0.011	0.042	0.048	0.033	0.026
std deviation	0.185	0.114	0.101	0.164	0.118	0.127	0.131
Sharpe ratio	0.302	0.984	0.161	0.342	0.457	0.323	0.264
I_{tr}	0.667	–	0.38	0.620	0.54	0.491	0.463

Table 3:
Optimization of State Space Model.The optimal values of the various parameters are given in the second colomn. The associated standard errors are computed analytically from the Hessian Matrix.

we also report some test statistics on the reliability of the rule. The first test statistic is the ideal rerturn ratio. It entails the ratio of the number of times the correct trading position was held against the position that would have maximised the accumulated returns.

$$I_{tr} = \frac{P_{tr}}{\overline{P}}$$

where $P_{tr} = 1$,if $R^{tr} = \max(R^m, R^b)$ and 0 otherwise.
$\overline{P}$ is the sample size.
The next statistic is the Sharpe ratio which is defined as the return from the rule per unit of standard deviation. The tests and statistics are reported in table 3:

4.3 Mean Reversion across time

The above model assumes that mean reversion occurs in the next period. Mean reversion may occur after more than one year. We take into account this possibility by assuming the following. If the rule states that the market is overpriced in a particular year, bonds will be held for a series of years, namely for 2,3 and 4 years respectively.

The results are conditional on the particular net present value approach that we have. Interestingly, none of the models appear to beat the passive Buy and Hold strategy in terms of the annual mean returns. However, the Sharpe ratio shows to use that most of

	BH	M1	M2	M3	M4	M5
2 years						
Mean returns	0.056	0.017	0.055	0.055	0.037	0.040
Standard deviation	0.185	0.103	0.162	0.124	0.129	0.122
Sharpe ratio	0.302	0.164	0.340	0.443	0.288	0.326
3 years						
Mean returns	0.056	0.012	0.060	0.053	0.040	0.043
Standard deviation	0.185	0.114	0.154	0.125	0.114	0.098
Sharpe ratio	0.302	0.107	0.391	0.425	0.352	0.437
4 years						
Mean returns	0.056	0.012	0.060	0.053	0.039	0.034
Standard deviation	0.185	0.115	0.153	0.122	0.131	0.108
Sharpe ratio	0.302	0.105	0.393	0.432	0.300	0.315

Table 4:
Mean returns over the different Holding periods. The optimal values of the various parameters are given in the second colomn. The associated standard errors are computed analytically from the Hessian Matrix.

the trading strategies tend to overpower the passive Buy and Hold over longer periods of time. This may be explained by the fact that there are times the asset is held in bonds. Bond returns express lower volatility than market returns. Returns from the different stategies, even after accounting for the lower risk, are not significantly higher than the buy and hold strategy. The excess returns of the trading strategies over the Buy and Hold may be negative due to transaction costs, which are not taken into account in this model.

Reversion towards the dynamic mean takes time in either model. This is best illustrated by the fact that the trading strategy is not more profitable. The annual returns from the simple dividend discount model does not increase as the number of holding periods increases. On the other hand, the strategy contrarian to the mean reverting model performs better. The remaining trading strategies based on reversion appear to have different mean reversion times, which is defined as the number of periods it takes for price to return close to the fundamental value.

We also perform an a replacement sampling experiment. The replacement sampling exercise is performed in order to deal with potential biasses that may occur. Random dates are chosen and

Horizon	M1	M2	M3	M4	M5
2 years	20	20	18	19	20
3 years	19	17	12	15	16
4 years	18	18	20	14	14

Table 5:
Replacement Sampling. The table relates to the score for the different strategies over the different holding horizons.The benchmark (Buy and Hold) for the experiment is 20.

the indicator variable will denote the decision to go long on bonds or equity. If the indicator variable predicts that the asset should be held in the index, then the index will be held for the horizon under investigation. The returns from the holding horizon is then compared to the Buy and Hold strategy. If the accumulated returns from the rule exceeds the Buy and hold for that period, a score of 2 is given. If the rule equals the passive Buy and Hold strategy, a score of one is given. If the passive Buy and Hold strategy exceeds the trading rule, then a score of zero is given. The results are presented in table...

The strategies tend to perform well for the 2 years horizons and marginally worse over the longer horizons. in stark contrast to the accumulated returns results, the first trading strategy tends to perform better over the different horizons. On the other hand, the other trading rules tend to show a pivotal behaviour with respect to the horizon involved. The second definition of the net present value, which tends to work well for the accumulated returns case, tend to perform marginally well over the 2 year horizon. None of the trading rules tend to overcome the passive Buy and Hold in the above.

5 Conclusion

This paper draws attention to a relatively passive trading rule that can be implemented by identifying whether equity indices are overpriced or underpriced. If the equity market is underpriced, the asset should be held long in the equity market and vice versa. A conceptual issue addressed in this paper is the valuation of the asset, which is dependent on the future dividends, expected returns and expected dividend growth rates. All these series are unidentifiable in real time. The series for the three variables were derived using both state space models and regression models.

The interesting finding of this paper is that the rule does work for long periods of time.The annual data results show that in real time, it may be difficult to identify and exploit mean reversion. The model is dependent on the defintion of the net present value. The econometrician's model and the structural model tend to be equally worse when compared to the passive buy and hold. The only advantage that this study exposes on the mean reverting rules is that they tend to exhibit lower risks. However, if transaction costs are taken into account the Sharpe ratio may be lower.and all of the models may be worse than the passive and buy strategy. However, the definition of the net present value which posits that both the growth rate of dividends and prices should be forecast. The hybrid model performs better, but still equals to the passive Buy and Hold on a four year holding horizon.

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A Appendix

A.1 The Net Present Value Model.

Equations 4 , 5 and 6 are represented

$$r_t = \log(\frac{P_{t+1} + D_{t+1}}{P_t}) \tag{27}$$

$$PD_t = \frac{P_t}{D_t} \tag{28}$$

$$\Delta d_{t+1} = \log(\frac{D_{t+1}}{D_t}) \tag{29}$$

The return process can be written as

$$\begin{aligned} r_t &= \log((\frac{P_{t+1} + D_{t+1}}{P_t}).\frac{D_t}{D_t}.\frac{D_{t+1}}{D_{t+1}}) \\ & \log((\frac{P_{t+1}D_t + D_{t+1}D_t}{P_t D_{t+1}}).\frac{D_{t+1}}{D_t}) & (30) \\ & \log((\frac{D_t}{P_t}.\frac{P_{t+1}}{D_{t+1}} + \frac{D_t}{P_t}).\frac{D_{t+1}}{D_t}) & (31) \\ & \log((\frac{P_{t+1}}{D_{t+1}} + 1)\frac{D_{t+1}}{D_t}.\frac{D_t}{P_t}) & (32) \\ & \log(1 + e^{pd_{t+1})}) + \Delta d_{t+1} - pd_t & (33) \end{aligned}$$

Assuming the log linearization of Campbell and Shiller (1988) the returns can be written as

$$r_t \simeq \log((1 + e^{pd_{t+1})}) + \frac{\exp(\overline{pd})}{1 + \exp(\overline{pd})} + \Delta d_{t+1} - pd_t$$

$$r_t = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - pd_t$$

Hence,

$$pd_t = \kappa + \rho pd_{t+1} + \Delta d_{t+1} - r_{t+1}$$

A.2 State Space Model.

In this section, we describe the Kalman filter procedure. The model has been coded using Ox 5. From the paper, there are two measurement equation and one transition equation. Equations 16, 17 and 18 can be written in this form:

$$X_t = FX_{t-1} + R\varepsilon_t$$

$$Y_t = M_0 + M_1 Y_{t-1} + M_2 X_t$$

where $Y_t = \begin{bmatrix} \nabla d_t \\ pd_t \end{bmatrix}$

The variables of the transition equation are $X_t$and $\varepsilon^x_{t+1}$and are made up of the following elements:

$$X_t = \begin{bmatrix} \widehat{g}_{t-1} \\ \varepsilon^D_t \\ \varepsilon^g_t \\ \varepsilon^\mu_t \end{bmatrix} \qquad F = \begin{bmatrix} \gamma_1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \varepsilon^x_{t+1} = \begin{bmatrix} \varepsilon^D_{t+1} \\ \varepsilon^g_{t+1} \\ \varepsilon^\mu_{t+1} \end{bmatrix}$$

The parameters of the measurement equation include parameters of the net present value model to be estimated. These are defined as :

$$M_0 = \begin{bmatrix} \gamma_0 \\ (1-\delta_1) * A \end{bmatrix} \qquad M_1 = \begin{bmatrix} 0 & 0 \\ 0 & \delta_1 \end{bmatrix} \qquad M_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ B_2(\gamma_1 - \delta_1) & 0 & B_2 & -B_1 \end{bmatrix}$$

The variance covariance matrix from the state space model is given by :

$$\Sigma = var \begin{bmatrix} \varepsilon^g_{t+1} \\ \varepsilon^\mu_{t+1} \\ \varepsilon^d_{t+1} \end{bmatrix} = \begin{bmatrix} \sigma^2_g & \sigma_{g\mu} & \sigma_{gd} \\ \sigma_{g\mu} & \sigma^2_\mu & \sigma_{D\mu} \\ \sigma_{gd} & \sigma_{D\mu} & \sigma^2_D \end{bmatrix}$$

The Kalman Filter procedure is given by the following equations :

$$
\begin{aligned}
X_{0|0} &= E[X_0] \\
P_{0|0} &= E[X_t X_t'] \\
X_{t|t-1} &= F X_{t-1|t-1} \\
P_{t|t-1} &= F P_{t-1|t-1} F' + R \Sigma R' \\
\eta_t &= Y_t - M_0 - M_1 Y_{t-1} - M_2 X_{t|t-1} \\
S_t &= M_2 P_{t|t-1} M_2' \\
K_t &= P_{t|t-1} M_2' S_t^{-1} \\
X_{t|t} &= X_{t|t-1} + K_t \eta_t \\
P_{t|t} &= (I - K_t M_2) P_{t|t-1}
\end{aligned}
$$

The parameters to be optimized are :

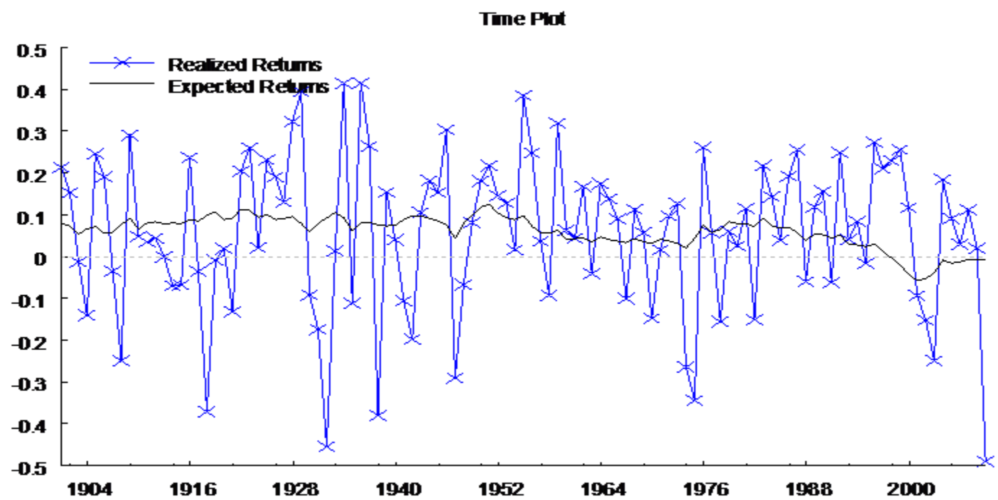
$$\Theta = (\gamma_0, \delta_0, \gamma_1, \delta_1, \sigma_g, \sigma_\mu, \sigma_D, \rho_{g\mu}, \rho_{gD}, \rho_{\mu D})$$

A.3 Other Statistical results

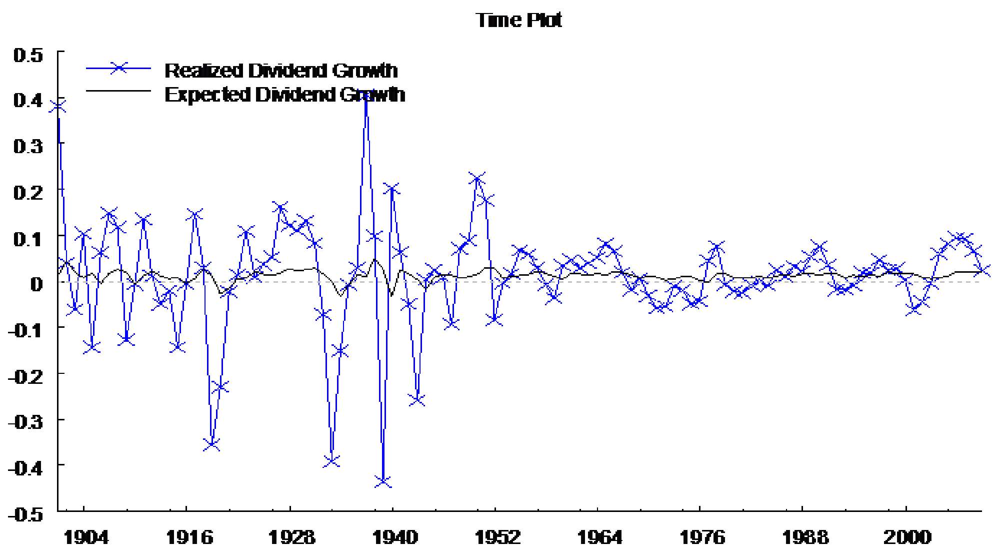
	$\mathbf{R}_t$	$\boldsymbol{\mu}_t$	$\boldsymbol{\Delta}\mathbf{d}_t$	$\mathbf{g}_t$
Test of I(0)				
Robinson-Lobato	$-0.92\{0.82\}$	$0.20\{0.42\}$	$-1.27\{0.90\}$	$-0.01\{0.50\}$
KPSS test	$0.04\{< 1\}$	$0.29\{< 1\}$	$0.03\{< 1\}$	$0.06\{< 1\}$
Lo's RS	$0.87\{< 0.9\}$	$0.82\{< 0.975\}$	$0.78\{< 0.99\}$	$0.51\{< 0.8\}$
Test of I(1)				
ADF	$-9.20\{< 0.01\}$	$-1.77\{< 0.9\}$	$-8.89\{< 0.01\}$	$-8.64\{< 0.01\}$
Phillips-Perron	$-9.30\{< 0.01\}$	$-1.79\{< 0.9\}$	$-8.92\{< 0.01\}$	$-8.72\{< 0.01\}$
DF-GLS	$-7.58\{< 0.01\}$	$-1.80\{< 1\}$	$-8.78\{< 0.01\}$	$-8.62\{< 0.01\}$
P	$1.18\{< 0.01\}$	$4.30\{< 1\}$	$1.26\{< 0.01\}$	$1.03\{< 0.01\}$

Table 6:
Tests of Stationarity on Expected Returns and Expected Dividend Growth: Sample 1900-2008. The Robinson- Lobato, KPSS and Range -scale test assume that the series is stationary under the null hypothesis. The rest of the statistics assume non stationarity. The figures in parentheses represent the p-values at which the null hypothesis is rejected.

A.4 Graphical Plots

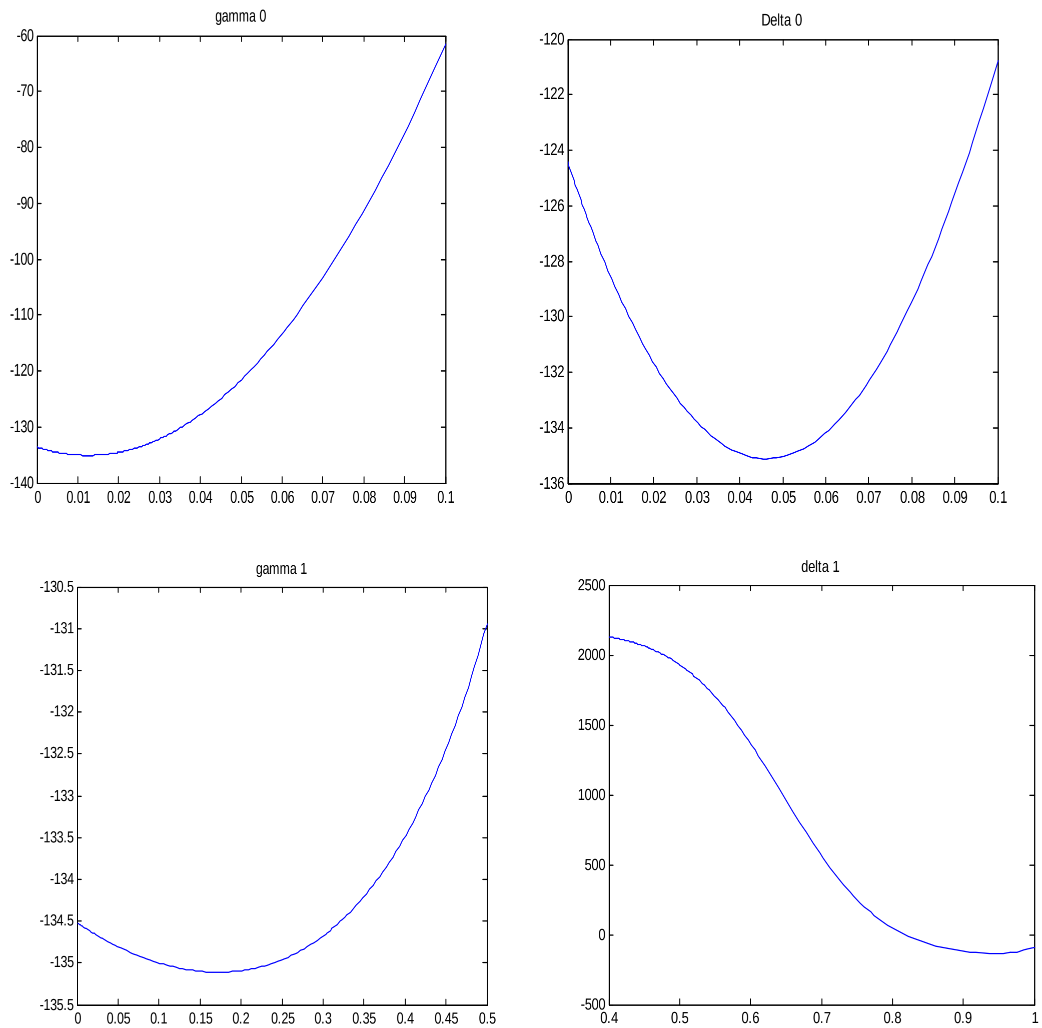


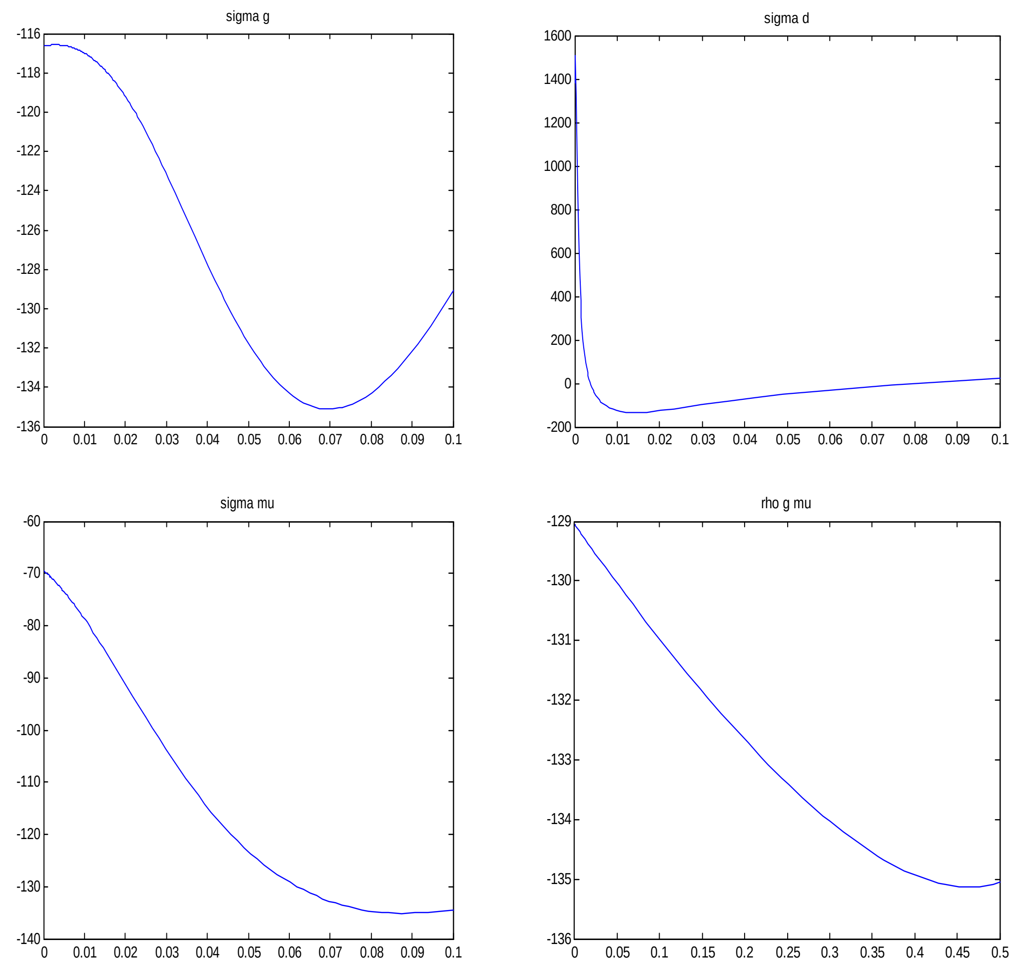
Plot of Realized and Expected Returns

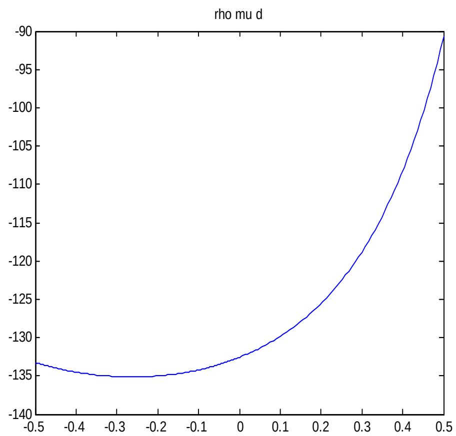


Plot of Annual Dividend Growth rate.

A.5 Sensitivity of optimal parameters




sigma g
-116
-118
-120
-122
-124
-126
-128
-130
-132
-134
-136
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
sigma d
1600
1400
1200
1000
800
600
400
200
0
-200
sigma mu
-60
-70
-80
-90
-100
-110
-120
-130
-140
rho g mu
-129
-130
-131
-132
-133
-134
-135
-136
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5


rho mu d
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-95
-100
-105
-110
-115
-120
-125
-130
-135
-140
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5