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Financial bubbles and sustainability
of public debt: The case of Spain

by

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Financial bubbles and sustainability of public debt: The case of Spain

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Abstract

In this paper the dynamics of the Spanish public debt-GDP ratio is analysed during the period 1850-2020. We use a recent procedure to test for recurrent explosive behaviour (Phillips, Wu and Yu, 2011, and Phillips, Shi and Yu, 2015a, 2015b) in order to identify episodes of explosive public debt dynamics and also the episodes of fiscal adjustments over this long period.

Keywords: Public debt; Fiscal sustainability; Explosiveness; Recursive unit root test; Spain; COVID-19

JEL classification: C12, C22, E62, H62, H63

1 Introduction

Since the start of the COVID-19 crisis, Spanish government has focused on doing whatever it takes to limit its consequences in economy and employment. The response of Spanish fiscal policy for an unprecedented crisis has

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included public health measures, unemployment benefits, wages subsidies, liquidity support for small-and medium-sized firms to prevent a wave of defaults and mass layoffs, and cash transfers to support the poor and informal workers and self-employed who lost jobs, among others. The massive fiscal support, provided since the start of the COVID-19 crisis, has succeeded in protecting people and preserving jobs. But it has considerably increased public expenditure and, together with sharp falls in tax revenues owing to the recession, it has pushed the Spanish public debt to a recent all-time high of 120.3 percent of GDP in 2020. In the eurozone (as in other advanced economies and some emerging market economies), European Central Bank purchases of government debt have helped to keep interest rates at historic lows and have supported government borrowing.

This recent international economic crisis, triggered by COVID-19 pandemic, and the attempt to alleviate it through Keynesian policies has put public budget figures in the red and it has turned the attention of governments back to the crucial issue of fiscal sustainability. The role of fiscal policy goes beyond the traditional stabilization function. Questions such as the balancing of budget deficits, the interactions between monetary and fiscal policies, and the fiscal discipline required in monetary unions, have also been intensively discussed over the last decades. In particular, one of the main problems concerning fiscal authorities is the sustainability of government deficits, which is related to the issue of long-run solvency.

Fiscal policy is regarded as sustainable when, if maintained in the indefinite future, it does not violate the solvency constraint; and a government is said to be solvent if the present value budget constraint, i.e., its intertemporal budget constraint (IBC) holds. In other words, the public deficit can be sustainable if the government can borrow. However, if the interest rate on the government debt exceeds the growth rate of the economy, debt dynamics would lead to an ever-increasing ratio of debt to GDP. The dynamics of debt accumulation could only be stopped only if the ratio of the budget deficit to GDP would turn to be a surplus, or if seigniorage were allowed for.

The empirical analysis of long-run fiscal sustainability will be performed for the case of Spain over the period 1850–2020. Recall that a common criticism to most tests of the IBC is that the econometric procedures used require a large number of observations. Accordingly, the use of a longer than usual span of data (i.e., 171 years) should allow us to obtain some more robust results than in most previous analyses. Moreover, the Spanish case can be of interest given the permanent difficulties experienced when balancing the

government budget across those years. Furthermore, the Spanish economy seems to be an interesting case study because it has been characterized by chronic government deficits and episodes with high levels of public debt. As far as we know, there are no empirical tests available in the literature regarding the sustainability of Spanish fiscal policies from a long-term perspective for such a long period.

In order to detect episodes of potential explosive public debt dynamics we use recent recursive unit root tests for explosiveness proposed by Phillips, Wu and Yu (2011, 2014), and Phillips, Shi, and Yu (2015a,b).

The scheme of the paper is as follows. Section 2 lays out the paper's connections with the literature. Next, in Section 3 we introduce the econometric methodology. Section 4 presents and discusses the main empirical results. Section 5 draws the main conclusions.

2   Literature review

2.1   The standard empirical analysis of the long-run fiscal sustainability

The sustainability of public finances, also referred to as fiscal sustainability, is the ability of government to sustain its current spending, tax and other policies in the long-run without threatening the government solvency or without defaulting on some of the government's liabilities. In other words, fiscal sustainability requires a government to be solvent, i.e., it has to be able to repay its debt at some point in the future.

In order to describe the possible ways of achieving fiscal sustainability, we will make use of the budget identity that links the public deficit to public revenues, public spending, and stock of public debt. The public deficit is the difference between public spending and public revenues. It also equals the change in public debt. In algebraic terms, let DEF_t denotes the total public deficit (i.e., including interest payment) in the year t, T_t total revenues, G_t the primary expenditures (i.e., excluding interest payment), B_{t-1} the stock of public debt at the end of year $t-1$ (all variables in nominal terms), and i_t the long-run interest rate. The budget identities are then,

$$DEF_t = G_t - T_t + i_t B_{t-1} \tag{1}$$

$$B_t = B_{t-1} + DEF_t \tag{2}$$

From equations (1) and (2), the nominal budget equation can be written as,

$$B_t = G_t - T_t + (1 + i_t)B_{t-1} = DEF_t^0 + (1 + i_t)B_{t-1} \tag{3}$$

where the primary public deficit $DEF_t^0 = G_t - T_t$.

The corresponding GDP-ratio version,

$$\frac{B_t}{Y_t} = \frac{DEF_t^0}{Y_t} + \frac{(1 + i_t)}{(1 + \gamma_t)} \cdot \frac{B_{t-1}}{Y_{t-1}} \tag{4}$$

where Y_t is the nominal GDP and $\gamma_t = Y_t/Y_{t-1} - 1$ is the nominal GDP-growth rate.

The equation (4) is similar in that it expresses period-t public debt as sum of a flow variable -the primary public deficit- and the previous period's public debt multiplied by a propagation factor.

Let b_t denote a generic, scaled version of public debt (e.g., the GDP-ratio, B_t/Y_t), let s_t denote the corresponding GDP-ratio version of the primary public surplus $(-DEF_t^0/Y_t)$, and let r_t denote the corresponding GDP-ratio version of the "return" on public debt, e.g., $r_t = (1+i_t)/(1+\gamma_t) - 1 \approx i_t - \gamma_t$.
[1]

The dynamic of public debt can be described compactly as,

$$b_t = (1 + r_t) \cdot b_{t-1} - s_t \tag{5}$$

From equation (5), we can readily compute the paths of public debt implied by arbitrary sequences of public primary surplus and interest payments. Iterating backward gives the following expression which mainly serves as the starting point for the theoretical analysis, with relevant empirical implications for fiscal sustainability, in Bohn (1998, 2007, 2008),

$$b_{t+n} = \left[\prod_{k=0}^{n} (1 + r_{t+k}) \right] \cdot b_{t-1} - \sum_{j=0}^{n} \left[\prod_{k=j+1}^{n} (1 + r_{t+k}) \cdot s_{t+j} \right] \tag{6}$$

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[1]Following Bohn (1998, 2007, 2008), we can also use alternative scale variables to GDP, like population, general price indexed, among others.

The derivation of conditions for fiscal sustainability is obtained from equation (6) in three steps. First, replace the returns on public debt in (6) by a fixed value r and take conditional expectations,

$$E_t\left[b_{t+n}\right] = (1+r)^n.b_t^* - \sum_{j=0}^{n}(1+r)^{n-j}.E_t\left[s_{t+j}\right] \qquad (7)$$

where $b_t^* = (1+r_t).b_{t-1}$ denotes public debt at the start of period t and where E_t denotes conditional expectations.

Second, divide by $(1+r)^n$ and rearrange to obtain,

$$b_t^* = \sum_{j=0}^{n} \frac{1}{(1+r)^j}.E_t\left[s_{t+j}\right] + \frac{1}{(1+r)^n}.E_t\left[b_{t+n}\right] \qquad (8)$$

Third, assume the discounted sum converges and take the limit $n \to \infty$,
t

en,

$$b_t^* = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j}.E_t\left[s_{t+j}\right] + \lim_{n\longrightarrow\infty} \frac{1}{(1+r)^n}.E_t\left[b_{t+n}\right] \qquad (9)$$

Equation (9), shows that initial public debt equals the expected present value of future primary public surpluses if and only if discounted future public debt converges to zero. That is,

$$b_t^* = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j}.E_t\left[s_{t+j}\right] \qquad (10)$$

and is equivalent to the current value of future public debt being convergent to 0,

$$\lim_{n\longrightarrow\infty} \frac{1}{(1+r)^n}.E_t\left[b_{t+n}\right] = 0 \qquad (11)$$

Equation (10) is commonly known as the Intertemporal Budget Constraint (IBC) and equation (11) as the Transversality Condition (TC) of the intertemporal decision problem of the government.

According to equation (10), the condition for fiscal sustainability requires that the government must run expected future budget surpluses equal, in

present-value terms, to the current value of its outstanding debt. The TC, rules out a Ponzi scheme (whereby debt is perpetually rolled over) as the necessary condition for lenders to hold government bonds. There is a large literature on the topic, though empirical tests of solvency (or fiscal sustainability), have gone through different stages.[2] Below are shown several methods that have been used in empirical applications to test whether this TC is fulfilled.[3]

2.2   Stationarity and linear cointegration

The usual procedure in most of the empirical contributions on the long-run sustainability of budget deficits consists of testing the government's IBC. It is a standard practice to examine the stationarity (with or without exogenous or endogenous structural changes) and linear cointegration (with or without exogenous or endogenous structural changes) of fiscal variables involved at IBC. The first studies, such as those of Hamilton and Flavin (1986) and Wilcox (1989), dealt with the IBC by focusing on the univariate properties (stationarity) of the public deficit and debt, but soon the linear cointegration approach became predominant. Trehan and Walsh (1988, 1991), Hakkio and Rush (1991), Haug (1991), Smith and Zin (1991) and Quintos (1995) analysed the presence of a long-run linear cointegration relationship between government revenues and expenditures.

These methods have been applied in previous analyses in several studies for testing the sustainability of Spanish fiscal policy. Camarero, Esteve, and Tamarit (1998) and de Castro and Hernández de Cos (2002) used linear cointegration with endogenous structural changes. Both studies pointed towards a gradual rather than a sudden change towards fiscal sustainability in the late eighties-early nineties in Spain, probably motivated by the beginning of a period defined by a greater effort to recover the control of public deficit.

More recently, Camarero, Carrion-i-Silvestre, and Tamarit (2015) used a cointegration model with regime shifts between public revenues and pub-

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[2]A very good, updated and clarifying study on the different approaches to evaluate this question is the one by D'Erasmo and Zhang (2016), where the authors identify the more important defaults in the traditional approach to evaluate debt sustainability, and examine three alternative approaches that provide useful econometric and model-simulation tools to analyze debt sustainability.

[3]For a recent review of empirical applications, see Beqiraj et al (2018) and the refferences therein.

lic expenditures, and between public debt and public expenditures. This methodology was applied to a group of 17 OECD countries, including Spain. For this country, they found evidence of strong fiscal sustainability only for the period 1989-2006 between revenues and expenditures, and evidence in favor of weak or ex post version of sustainability and, therefore, that Spanish governments were compelled to adjust their fiscal policies in order to fulfill their long-run budgetary constraint.[4] [5] [6]

2.3   Fiscal reaction function

Bohn (2007) published a harsh critique against the above mentioned procedures (unit root and cointegration conditions) describing them as assessing sustainability as a mere "mechanical exercise". Bohn (2007) demonstrated that test of the IBC, based on unit root and cointegration tests, are incapable of rejecting fiscal sustainability. More specifically, Bohn (2007, 2008) proved that the public debt series integrated of any finite order satisfies the sustainability condition (11). Fiscal unsustainability would implies explosive debt dynamics. However, the standard unit root and cointegration tests, by construction, do not consider explosive dynamics as a hypothesis, and these test only discriminate between different orders of integration, i.e., between I(0) (stationary) and I(1) (nonstationary, integrated of order 1) processes.

In a companion paper, Bohn (1998) goes one step further, arguing that tests based purely on time-series properties of net foreign assets and net exports miss the general equilibrium conditions linking external balance to the rest of the economy. Bohn's "model-based-sustainability" framework suggests estimating econometrically the conditional relationship between net foreign assets and the net exports. For him, error correction-type policy reaction functions are more promising for understanding deficit problems. Moreover, the cointegration relationship between government and expenditures and imports is not a necessary condition for the no-Ponzi game condition

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[4]For the formulation and development of strong and weak conditions of fiscal sustainability, see Quintos (1995).

[5]For an application with international data, see Afonso (2005). In order to assess the sustainability of budget deficits, he uses cointegration tests between public expenditures and public revenues, allowing for structural breaks for the EU-15 countries for the 1970-2003.

[6]Other alternative empirical procedures of analysis can see in Westerlund and Prohl (2010). They use panel cointegration tests of the sustainability hypothesis in rich OECD countries.

to hold but that a long-run relationship under as an error-correction specification between primary public deficit and public debt has to be fulfilled in order to avoid any explosive outcome among the variables that determine the fiscal equilibrium in the long-run.

This procedure, based on a fiscal policy reaction function, has been applied in previous analyses on the sustainability of Spanish fiscal policy. First, Bajo-Rubio, Díaz-Roldán, and Esteve (2009, 2014) used a modification of Bohn's reaction function based on the Fiscal Theory of the Price Level (FTPL) and since the condition of fiscal solvency was satisfied, government deficit would have been sustainable along the sample period. Second, Escario, Gadea, and Sabaté (2012) used a multicointegration methodology that allowed the estimation of dynamic equilibrium relationships between flow and stock fiscal variables. The results reveal that seigniorage was essential in guaranteeing long-run government solvency.

Third, Camarero, Carrion-i-Silvestre, and Tamarit (2015) also used a multicointegration with structural breaks methodology that allowed the estimation of dynamic equilibrium relationships between flow and stock fiscal variables. For Spain, they found not evidence of multicointegration between public debt and real public expenditures and, therefore, the hypothesis of a strong fiscal sustainability (a second layer of fiscal sustainability) is rejected.

More recently, Paniagua, Sapena, and Tamarit (2017) proposed a time-varying estimate the fiscal reaction function to test long-term fiscal sustainability and the variation in the degree of responsiveness towards sustainability. This approach was applied a 11 Eurozone country panel (including Spain) through the Kalman Filter with a transition. The results show that Spanish economy does not exhibit a permanent fiscal reaction component but exhibit a salient time-varying component, i.e., has not been reacting in a systematic way to debt accumulation and only shows isolated episodes of fiscal reaction to public debt under extreme circumstances.

2.4 A time-varying fiscal policy

Fiscal sustainability, by definition, is a long-run condition. However, there is the possibility of some different patterns in the behavior of public debt for different subsamples or subperiods of such a full long sample. Thus, given that the concept of sustainability is based on a particular type of behavior of the time series in the long-run, what can happen is that there are a number of periods of relatively short duration displaying a different pattern, in this

case identified with explosive regimes. So, the fiscal sustainability, which is found in such analysis, can be viewed as a result of fluctuating fiscal policies. If short-run active fiscal policies, which are unsustainable in the long run, are followed by timely fiscal adjustments, then the long-run sustainability is not rejected. Therefore, suitable fiscal adjustments might bring the debt dynamics back to a sustainable path.

The traditional methods of stationarity and cointegration analysis (without endogenous structural changes) or methods based on a time-invariant reaction function are not designed for modeling transitory unsustainable dynamics of public debt. In this context would be justified a time-variant fiscal policy. Fiscal policies would change in response to economic, political and institutional environment. A large amount of papers on theoretical literature followed this approach (see, among others, Alesina & Drazen, 1991; Alesina & Perotti, 1995; Alesina et al., 1996; Alesina & Ardagna, 1998; Alesina et al., 1998; Perotti, Strauch, & von Hagen, 1998; Perotti, 1999; Wyplosz, 2005).

This approach based, on a time-variant fiscal policy, has been applied with non-linear effects in previous analyses on the sustainability of Spanish fiscal policy. First, Bajo-Rubio, Díaz-Roldán, and Esteve (2004) used a threshold autoregressive (TAR) model. They found strong evidence of non-linearities in the evolution of the Spanish budget deficit in terms of a threshold autoregressive model, so that the deficit dynamics would be different depending on whether the change in the deficit was below or above an endogenously estimated threshold; in other words, mean-reverting dynamic behavior in the budget deficit would be expected once such threshold was reached. Thereby, the fiscal sustainability has been attained due to the non-linear behavior of fiscal authorities. Second, Bajo-Rubio, Díaz-Roldán, and Esteve (2006) applied a threshold cointegration model. The results suggested the presence of significant non-linear effects in Spanish fiscal policy, so that fiscal authorities would cut deficits only if they are 'large', which would assure in turn their long-run sustainability. Finally, Bajo-Rubio, Díaz-Roldán, and Esteve (2010) employed a threshold cointegration model and a cointegrated regression model with multiple structural changes. The results showed that the government deficit has been strongly sustainable in the long-run, and fiscal sustainability has been attained due to the non-linear behavior of fiscal authorities.[7]

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[7]See Arestis et al (2004), Chortareas et al. (2008) or, more recently, Tran (2018), with applications to international data.

3 Econometric methodology

3.1 A model for recurrent explosive behavior in time series data

Evans (1991) argued that standard right-tailed unit root tests, frequently used to evaluate long-run fiscal sustainability, when applied to the full sample, have little power to detect periodically collapsing bubbles (the explosive behavior is only temporary) and demonstrated this effect in simulations. The low power of standard unit root tests is due to the fact that periodically collapsing bubble processes behave rather like an $I(1)$ process or even a stationary linear autoregressive process when the probability of bubble collapse is non-negligible, thereby confounding empirical evidence.[8]

To overcome the problem identified in Evans (1991), Phillips, Wu and Yu (2011, PWY henceforth) and Phillips, Shi, and Yu (2015a, 2015b, PSY henceforth) developed a new recursive econometric methodology for real-time bubble detection that proved to have a good power against mildly explosive alternatives. The interest in testing algorithm is whether a particular set or group of consecutive observation comes from an explosive process (H_A) or from normal martingale behavior (H_0). The algorithm testing is based on a right-tailed unit root test proposed by Phillips, Shi and Yu (2014).

On the one hand, the martingale null is specified as,

$$H_0 : y_t = kT^{-\eta} + \delta y_{t-1} + \varepsilon_t \tag{12}$$

with constant k and $\eta > 1/2$, and where y_t is data series of interest (in our case the public debt) at period t, ε_t is the error term, and T is the total sample size.

The hypothesis that the parameter $\delta = 1$ implies that y_t is integrated of order one, i.e., $y_t \sim I(1)$.

On the other hand, the alternative is a midly explosive process, namely,

$$H_A : y_t = \delta_T y_{t-1} + \varepsilon_t \tag{13}$$

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[8]An illustrative pedagogical introduction to the empirical analysis of searching for collapsing bubbles in nonstationary time series, and its theoretical foundations, can be found in Phillips (2012). Other relevant references are the seminal papers by Yu and Phillips (2009) and Phillips and Yu (2011).

where $\delta_T = (1 + cT^{-\alpha})$ with $c > 0$ and $\alpha \in (0,1)$, and it must be indicated that this type of mildly explosive and collapsing behavior under the alternative hypothesis corresponds to, at least, one subperiod of the full sample, not to the whole sample. In this case, if $\delta_T > 1$, it implies the explosive behavior of y_t over sub-period $t \in [T_1, T_2]$.[9]

In addition to the classic reference of Evans (1991), Charemza and Deadman (1995) extends the above analysis to the case of multiplicative processes with a stochastic explosive root encompassing non-negative processes used in the analysis of exuberant time series. The formulation of equation(12), as a restrictive representation of the generating process under the null hypothesis, includes a particular, not standard, representation for the drift term. Given that the recursive representation can be written as,

$$\frac{1}{\sqrt{T}}y_t = kT^{1/2-\eta}\left(\frac{t}{T}\right) + \frac{1}{\sqrt{T}}y_0 + \frac{1}{\sqrt{T}}\sum_{j=1}^{t}\varepsilon_j \qquad (14)$$

where $T^{1/2-\eta} \longrightarrow 0$ as $T \longrightarrow \infty$, so that the drift term is asymptotically negligible and does not interfere with the standard asymptotics for a nonstationary process.

3.2 Recursive unit root test for explosiveness

The methodology developed in PWY and PSY can be applied to test the unit root hypothesis in the standard model described in (12) against an alternative of multiple sub-periods of explosive behavior $[T_1^{(i)}, T_2^{(i)}], i = 1, 2, ...k, k \geq 1]$, where of the public debt dynamics is described in (13). The sustainable dynamics of public debt implies that y_t is a process integrated $I(1)$ that is interrupted by recurrent episodes of explosive public debt dynamics. That is, it represents the maintained hypothesis of the empirical analysis in order to obtain empirical evidence in favor of a sustainable public debt process in terms of a "global" nonstationary sequence eventually interrupted by, at least, one collapsing mildly explosive episode.

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[9] For the formulation and development of asymptotics for this type of mildly integrated (when $c < 0$) and mildly explosive (when $c > 0$) behavior, see the basic references to the works of Phillips and Magdalinos (2007a, b).

The testing procedure is developed from a regression model of the form,

$$\Delta y_t = \beta_0 + \beta_1 y_{t-1} + \sum_{i=1}^{K} \lambda_i \Delta y_{t-i} + \varepsilon_t \qquad (15)$$

where β_0, β_1, and λ_i are model coefficients, K is the lag order, and ε_t is the error term. The key parameter of interest is β_1. We have $\beta_1 = 0$ under the null and $\beta_1 > 0$ under alternative. The model is estimated by Ordinary Least Squares (OLS) and the t-statistics associated with the estimated β_1 is referred to as ADF statistic.

First, PWY proposed a sup ADF ($SADF$) statistic to test for the presence of explosive behavior in a full sample. In particular, the test relies on repeated estimation of the ADF model on a forward expanding sample sequence, and the test is obtained as the sup value of the corresponding ADF statistic sequence. In this case, the window size (fraction) r_w expands from r_0 to 1, where r_0 is the smallest sample window width fraction (which initializes computation of the test statistic) and 1 is the largest window fraction (the total sample size) in the recursion. The starting point r_1 of the sample sequence is fixed at 0, so the endpoint of each sample (r_2) equals r_w and changes from r_0 to 1. The ADF statistic for a sample that runs from 0 to r_2 is denoted by $ADF_0^{r_2}$.

The $SADF$ test is then a sup statistic based on the forward recursive regression and is simply defined as,[10]

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2} \qquad (16)$$

Second, PSY developed a double-recursive algorithms that enable bubble detection and consistent estimation of the origination (and termination) dates of bubble expansion and crisis episodes while allowing for the presence of multiple structural breaks within the sample period. They showed that when the sample includes multiple episodes of exuberance and collapse, the PWY procedures may suffer from reduced power and can be inconsistent, thereby failing to reveal the existence of bubbles. This weakness is a particular drawback in analyzing long time series or rapidly changing of data where more than one episode of explosive behavior is suspected.

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[10]This notation highlights the dependence of on $SADF$ of the initialization parameter r_0.

To overcome this weakness and deal with multiple breaks of exuberance and collapse, PSY proposed the backward sup ADF ($BSADF$) statistic defined as the sup value of the ADF statistics sequence over interval $[0, r_2 - r_0]$. That is,

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2} \tag{17}$$

where the endpoint of each sub-sample is fixed at $T_2 = [r_2 T]$ where $r_2 \in [r_0, 1]$, and the start point of each sub-sample, $T_1 = [r_1 T]$ varies from 1 to $T_2 - T_0 + 1 (r_1 \in [0, r_2 - r_0])$. The corresponding ADF statistics sequence is $\left\{ ADF_{r_1}^{r_2} \right\}_{r_1 \in [0, r_2 - r_0]}$.

PSY also proposed a generalized version of the sup ADF ($SADF$) test of PWY, based on the sup value of the $BSADF$. That is,

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1]} BSADF_{r_2}(r_0) \tag{18}$$

The statistic (18) is used to test the null of a unit root against the alternative of recurrent explosive behavior, as the statistic (16). It is important to note, and it must be clearly stated, that the fact that the two sequential versions of the ADF test indicated in equations (16) and (18) as the sup values in the sequences of the subsamples implies that all these tests are right-tailed, i.e., the rejection is obtained for large positive values. Moreover, it is relevant for these testing procedures the consistent estimation of the initialization and burst time periods of the explosive behavior when the null hypothesis is rejected.[11] [12]

The origination date $[T\hat{r}_e]$ of an episode of explosive behavior is defined as the first observation whose backward sup ADF exceeds the corresponding critical value,

$$\hat{r}_e = \inf_{r_2 \in [r_0, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) < scv_{r_2}^{\alpha_T} \right\} \tag{19}$$

where $scv_{r_2}^{\alpha_T}$ is the $100(1 - \alpha_T)$ % critical value of sup ADF statistic based on $[T r_2]$ observations and α_T the significance level which may depend on the

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[11] For more details of these recursive and sequential testing procedures can be found, for example and among some others, in Phillips and Shi (2020).

[12] The more recent and complete study on the properties of these estimates, both for the ADF-based detector and also for a CUSUM-type detector, and for different locations of the explosive sequence along the sample, can be found in Kurozumi (2021).

sample size T.

The termination date $[T\hat{r}_f]$ of an episode of explosive behavior is computed as the first observation after $[T\hat{r}_e] + \delta \log(T)$ whose sup ADF statistic falls below the corresponding critical value,

$$\hat{r}_f = \inf_{r_2 \in [\hat{r}_e + \delta \log(T)/T, 1]} \left\{ r_2 : BSADF_{r_2}(r_0) < scv_{r_2}^{\alpha_T} \right\} \tag{20}$$

where $\delta \log(T)$ is the minimal duration of an episode of explosive behavior.

4 Empirical application

4.1 Historical Fiscal data

We consider a long historical time series in which many fiscal crisis events are known to have occurred. The length of this database makes it particularly suitable for the econometric approach adopted in this paper (171 years).

The data and sources are:

- 1850-2000: a) public debt, total outstanding liabilities, B_t, from Carreras and Tafunell, X. (2005), Table 12.34, serie 2895; b) nominal GDP, Y_t, from Carreras and Tafunell, X. (2005), Table 17.7, serie 4744; c) the public debt-to-GDP ratio, $b_t = B_t/Y_t$.

- 2001-2020; d) public debt, general government, debt compiled according to Excessive Deficit Procedure (EDP), from Banco de España (2021a), Table, 2.15.a; e) nominal GDP, Y_t, from Banco de España (2021a, 2021b), Table 1.1; f) the public debt-to-GDP ratio (EDP). f) the public debt-to-GDP ratio, $b_t = B_t/Y_t$.

In our empirical analysis, we use annual data of the Spanish public debt-GDP ratio, b_t, for the period 1850-2020. Given that the present analysis is going to focus on the Spanish case, and that there is no previous studies on the solvency of the Spanish government in such a long-run period, we think that it is necessary to sketch the historical budgetary background. We can broadly follow dynamics of the path the Spanish public debt, as % of GDP, between 1850 and 2020 in Figure 1, where the expansions of public debt are markedly visible in it.

The Spanish case can be of interest given the permanent difficulties experienced when balancing the government budget across the sample. For most of this period, and until the fiscal reform of 1978, public revenues proved

insufficient to finance even small amounts of public expenditures, so public deficits became chronic. The immediate consequence of budget deficits was a huge increase in public debt.

The first historical public debt cycle took place between 1868-1881, which included a period of political instability after the so-called "Glorious Revolution" and "the revolutionary years" (1868-1874), the first Cuban War (1868-1878), and the second Cuban War (1879-1880). The maximum levels of government debt can be found at 1876 when it amounted until the 169% of GDP, the highest of the full sample. The second historical public debt cycle spanned from 1894 to 1909, covering the third Cuban War or Cuban War of Independence (1895-1898), the Philippine War (1896-1898), and the Spanish-American War of 1898. The peak level of government debt in this period can be found at 1902 when it amounted until the 128% of GDP. The third cycle covered the period 1933-1945. After that, a brief stage covering the austerity and rescheduling program implemented by Minister Fernandez-Villaverde in 1899 up to the First World War, the public deficits reappeared and became the norm again, and the public debt increased again. They gradually increased throughout a period of rising instability, until the Spanish Civil War broke out. Military spending during the conflict was undoubtedly substantial. The postwar consequences were clearly visible during Franco's autarky (the forties), again in the form of sizeable public deficits with the consequent increase of the indebtedness. As a consequence, the public debt-to-GDP ratio climbed to reach a peak of 71.8% in 1940.

The fourth historical public debt cycle took place under democracy with increasing but sustainable percentages until the fiscal crisis of 1992-1993 fiscal crisis when the debt-to-GDP ratio increased considerably again. The increase in public debt occurred due to expansionary fiscal policies necessary to finance infrastructures of the Universal Exposition of Sevilla and the Barcelona Olimpic Games in 1992. As a result, the public debt-to-GDP ratio increased to a peak of 64.7% in 1996. The fifth historical cycle of public debt took place in the period 2009-2014, after the Great Recession. It was a period of marked general decline (recession) observed in national economies worldwide that occurred between 2007–2009, originating in the United States.

The main cause of Spanish crisis was the housing bubble and the unsustainably high GDP growth rate. In this way, during the expansion of the Spanish economy in the period 1995-2007 (with an average increase in nominal GDP of 3.4%) prior to the international financial crisis of 2007-2008, against an international background marked by low interest rates, optimistic

expectations about economic growth and underpriced risks, the debt levels of households and non-financial corporations in the advanced economies increased markedly. In Spain this phenomenon became notably acute and debt ratios higher than those observed in other Eurozone countries were recorded. Single currency membership prompted an upward revision of expected incomes and set in place very loose financing conditions, the expansionary impulses of which on lending were not sufficiently countered by other economic policies. After 2008, the public debt-GDP ratio began an upward trend up reaching a new peak of around 100% in 2014.

On the one hand, the expansion was accompanied by a progressive concentration in transactions linked to the real estate market. Given the insufficiency of domestic saving to finance this expansion in lending, banks resorted to international debt markets. In this period, the Spanish economy recorded current-account deficits of a size unprecedented in the historical time series. Thus, the negative current-account deficit over GDP ratio was gradually increased from -4.3% to -9.4% in 2007. As a consequence, the financing of these deficits raised Spain's external debt to a very high level. Thus, the negative net foreign assets over GDP ratio increased by 62.7 percentage points of GDP between 2000 and 2009. Definitely, the wider debit balance vis-à-vis the external sector mainly reflects the Spanish economy's recourse to foreign savings to finance expansion of private investment. As a result of this, the net foreign assets climbed to reach a historical peak of -97.6% over GDP in 2009.

On the other hand, during and after the global financial crisis, falling house prices, and a tightening of collateral constraints for Spanish borrowers contributed to a sharp reduction in capital inflows, and to the persistent slump in Spanish real activity. As consequence, the Spanish current account balance has undergone a sharp adjustment and improved notably. Specifically, there was a correction of some 11.4 percentage points of GDP between 2007 and 2016, until reaching a surplus of 3.2%.

Finally, Figure 1 displays the behavior of the debt-GDP ratio in 2020. Since the beginning of the COVID-19 crisis, the Spanish government has focused on doing everything possible to limit its consequences on the economy and employment. The massive fiscal support, provided since the beginning of the COVID-19 crisis, has succeeded in protecting people and preserving jobs. But it has considerably increased public expenditure and, together with sharp falls in tax revenues owing to the recession, it has pushed the Spanish public debt to a recent all-time high of 120.3 percent of GDP in

2020, near the level it reached in 1902 during the war period Cuba and Philippines. The European Commission is pessimistic about the effect of the COVID-19 on the Spanish public debt/GDP ratio for the period 2021-2022, and expects it to remain around an average level of 124% (projections of European Commission, 2020).

4.2   Main results

The methodology developed in PWY and PSY was originally proposed to test for recurrent explosive behavior for U.S. stock market. As far as we know, part of this methodology has only been used to test explosive behavior of the public debt in two previous papers. Yoo (2012a) applied the approach proposed by PWY to test fiscal sustainability in the U.S. for the period 1791 to 2009. He show that the U.S. public debt-GDP ratio was explosive in nature during the sample period and that the World War II is responsible for this result. Yoon (2012b), which uses the same empirical procedures with quarterly data for the U.S. budget deficit and for the period 1947:Q1-2007:Q3, finds some evidence that the postwar U.S.budget deficit was explosive. Bystrov and Mackiewicz (2020) applied the procedure proposed by PSY in order to identify episodes of explosive debt dynamics in Sweden, the UK and the U.S. for the period 1792 to 2012. They find evidence of recurrent explosive behavior of public debt in the three advanced economies.

In this paper, we use the methodology developed in PWY and PSY to examine whether the Spanish public debt-GDP ratio has speculative bubble behavior at any point time for the period 1850-2020.

For our empirical application, the lag order K is selected by Bayesian information criterion (BIC) with a maximum lag order of 6, as suggested by Campbell and Perron (1991). We set the smallest windows size according to the rule $r_0 = 0.01 + 1.8/\sqrt{T}$ recommended by PSY, giving the minimal length of a sub-sample at 25 years. The origination (termination) of an explosive episode is defined as the first chronological observation whose test statistic exceed (goes below) its corresponding critical value.

Table 1 reports the $SADF$ and $GSADF$ tests of the null hypothesis of a unit root against the alternative of an explosive root to the public-debt-GDP ratio. The various critical values for each of the two test are also reported. We conduct a Monte Carlo simulation with 2,000 replications to generate the $SADF$ and $GSADF$ statistics sequences and the corresponding critical values at the 10, 5 and 1 per cent levels.

As can be seen in Table 1, we reject the unit root null hypothesis in favour of the explosive alternative at the 10% significance level for the $SADF$ test and the 5% significance level for $GSADF$ test. Both tests exceed their respective 10% and 5% right-tail critical values, giving any evidence that the public debt-GDP ratio had explosive subperiods. Consequently, we can conclude from both summary tests that there is some evidence of bubbles in the public-debt-GDP ratio.

Next, we use for the public debt-GDP ratio the PSY strategy. As noted in Phillips and Shi (2017, 2018), the PSY procedure also has the capability of identifying market downturns, in our case, fiscal adjustments.

To locate the origin and conclusion of the explosive public debt behavior and the fiscal adjustments episodes, Figure 2 plots the profile of the GSADF statistic for the public-debt-GDP ratio. We compared the $GSADF$ statistic with the 95% $SADF$ critical value for each observation of interest. The initial start-up sample for the recursive regression covers the period 1850-1873 (14% of the full sample). Figure 2 identifies of episodes of explosive public debt behavior and it permits to date-stamp its origination and termination, as well as the fiscal adjustments.

The first episode dated in 1874-1880, period of explosive behavior is related to the first and second Cuba wars and the budget efforts made in this period. The public-debt-GDP ratio remained around an average level of 150%, the highest level of entire of the sample. The second episode, occurred in 1917-1920, was a fiscal adjustment that occurred after an explosive debt path associated with Cuban War of Independence, the Philippine War, and the Spanish-American War (another pick of public debt was reached in 1902).The public debt was stabilized (the public-debt-GDP ratio was gradually decreased from 70% to 44.4%) after the austerity and rescheduling program implemented by Minister of Finance Fernandez-Villaverde in 1899 up to the First World War.

The third episode, dated between 1951-1981, is another fiscal adjustment and is correlated with the period of Franco's regime until the arrival of democracy in 1979. Both the absence of bonds issues during the Franco's mandate and the steady economic growth of the late 1950s and 1960s reduced the public debt/GDP ratio, so that during the Franco's era there was no formal public debt crisis. This result is clearly due to the stance of fiscal policy and to a policy of partial debt repudiation of the debt undertaken after the Spanish Civil War. In this period the public-debt-GDP ratio was gradually decreased from 46.2% to 19.4%.

The fourth episode of explosive behavior of public debt behavior between 1982-2002 was the result of chronic government deficits. Spain had to wait until the restoration of democracy in the late 1970s, and especially to integration into the now European Union (EU) in 1986, to enjoy a public sector comparable to the rest of Western Europe. Though both revenues and expenditures remained at low levels, the former were insufficient to finance even small amounts of the latter, so budget deficits dominated over most of this period. The immediate consequence of budget deficits was a huge increase in public debt (the debt-to-GDP ratio rose from 19.4% to 51.1%). The latest episode detected is a period of explosive debt behavior (2010-2020) is associated with the deep economic recession of 2008-2013 in the aftermath of the international financial crisis of 2007-2008, and the negative budgetary consequences (current and futures) of the recent international economic crisis triggered by COVID-19 pandemic in 2020.

Next, we use for the public debt-GDP ratio the PWY strategy. Figure 3 plots the SADF test against the corresponding 95% critical value sequence. We see that the PWY strategy identifies only the episodes related to the first and second Cuban wars. The estimated origination and termination dates for this episode are precisely the same as those of the PSY strategy (1874-1880). These results are not surprising since PSY proved the greater discriminatory power of the GSADF strategy found in the simulations and evidenced in the asymptotic theory. The new date-stamping strategy of PWY identifies all the historical episodes of explosive public debt behavior and also the fiscal adjustments episodes during this long period, whereas the recursive SADF procedure seems more conservative and locate fewer episodes of explosive behavior and of market downturns, in our case, fiscal adjustments.

Regarding to the differences in the results of the forward and backward versions of the sequential ADF tests, these can be explained by their power performance depending on the relative location of the sample size of the explosive behavior periods, and even the number of time periods that are involved in these episodes.

5   Conclusions remarks

In this paper, the dynamics of the Spanish public debt-GDP ratio is analysed during period 1850-2020. The longer than usual span of the data should allow us to obtain some more robust results than in most of previous analyses of

long-run sustainability.

We use recent procedures of testing for recurrent explosive behavior (Phillips, Wu and Yu, 2011, and Phillips, Shi and Yu, 2015a, 2015b) in order to identify episodes of explosive dynamic of public debt, which can be attributed to active budget policies (unsustainables) that ran in the past.

The new date-stamping strategy of Phillips, Shi and Yu (2015a, 2015b) identifies all historical episodes of explosive behavior of public debt and also the fiscal adjustments episodes over this long period, whereas the strategy developed by Phillips, Wu and Yu (2011) seems more conservative and locates fewer episodes of explosive behavior and of market downturns, in our case, fiscal adjustments.

The identified episodes of explosive behavior of public debt coincided with fiscal stress events, whereas fiscal adjustments, and changes in economic policies, stabilized public finances after periods of explosive dynamics of public debt.

6   Acknowledgements

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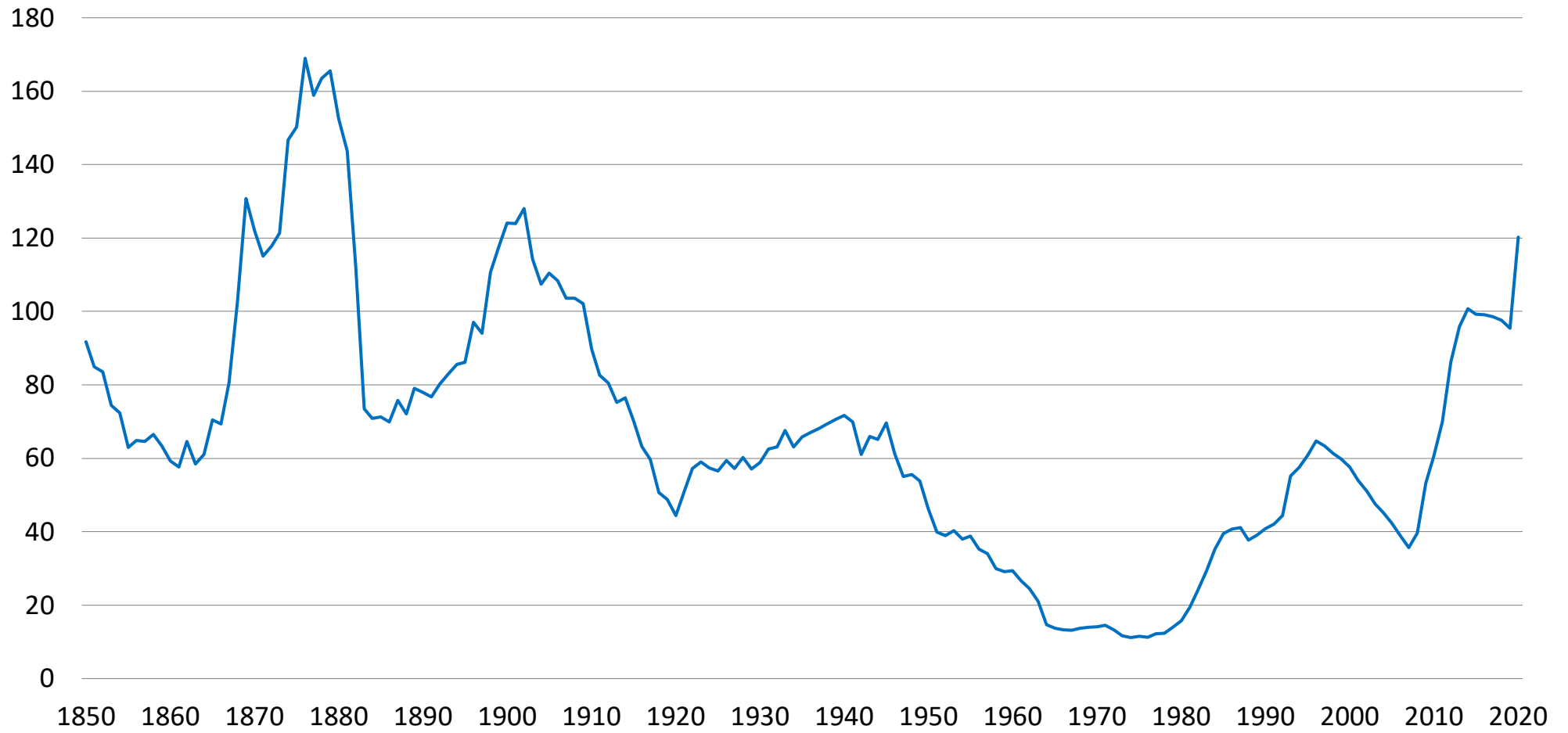
Table 1

Testing for explosive behavior in Spanish public debt-GDP ratio from 1850 to 2020

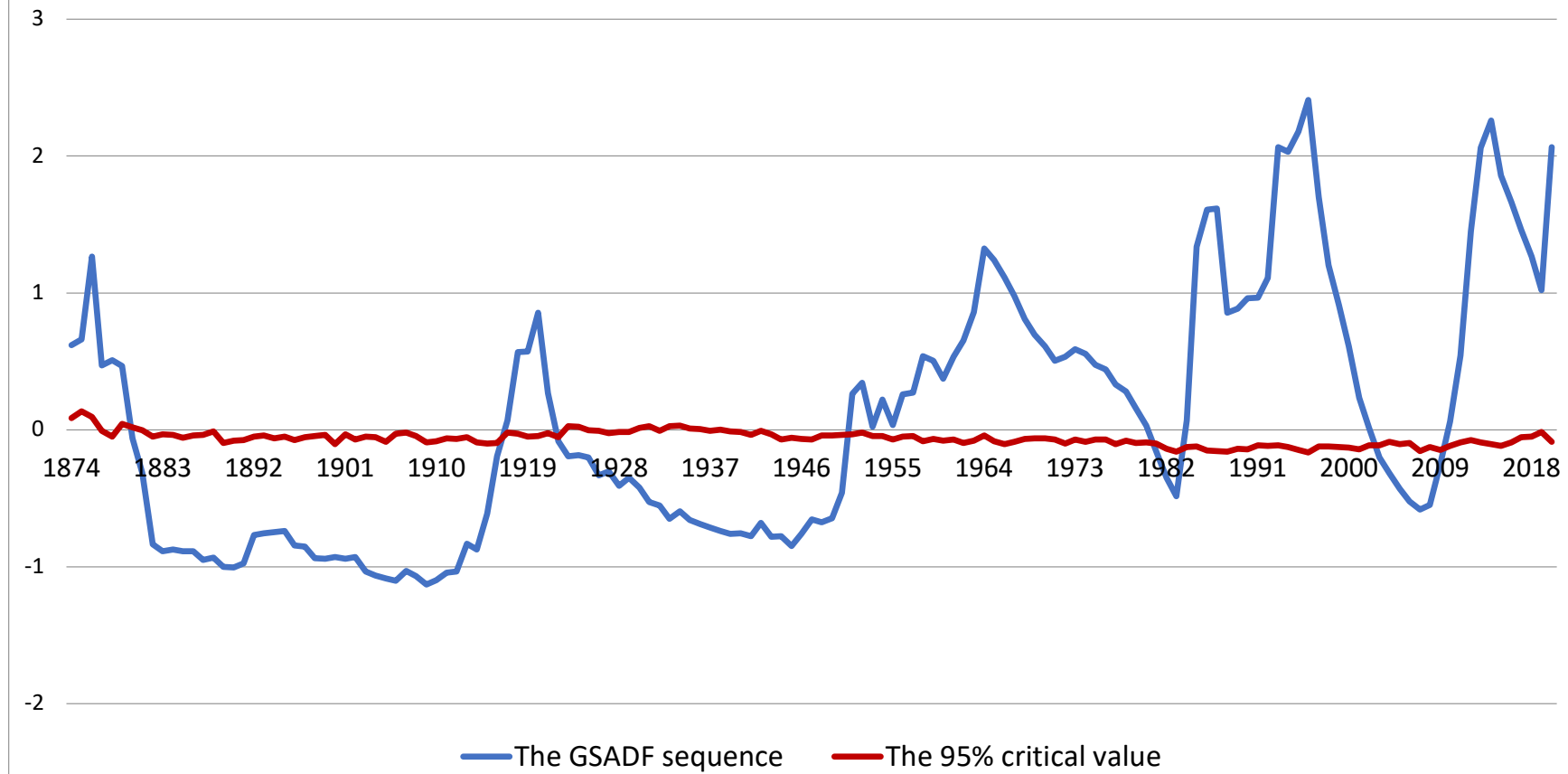| Unit root tests	Estimated Value	Finite Critical Value		
		1%	5%	10%
$SADF$	1.231***	1.937	1.322	1.082
$GSADF$	2.409**	2.717	2.071	1.831

Notes:

*, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively.

Figure 1
Spanish public debt as % of GDP
1850-2020

Figure 2
Date-stamping bubble periods in the Spanish public debt as % of GDP:
The GSADF test

The GSADF sequence —— The 95% critical value

Figure 3
Date-stamping bubble periods in the Spanish public debt as % of GDP:
The SADF test