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Testing public debt sustainability with time-varying volatility: the case of Italy, 1861-2024

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Abstract

This paper investigates the long-run dynamics and sustainability of the Italian public debt-to-GDP ratio from 1861 to 2024. To address the presence of non-stationary volatility, we employ the recently developed Time-Transformed Test methodology proposed by Kurozumi, Skorobotov, and Tsarev (2023). By utilizing the cumulative variance profile this approach homogenizes volatility across the time domain, ensuring the asymptotic validity of the STADF and GSTADF statistics and overcoming the size distortions inherent in standard recursive unit root tests. Our empirical findings confirm multiple explosive 'bubble' episodes in Italy's fiscal history, specifically identifying three critical periods of exuberance: the 1913 anticipatory wartime shock, the 1978–1998 structural imbalance, and the 2020 exogenous pandemic-induced spike. While the 1978–1998 episode represents a unique two-decade era of structural divergence sustained by high private savings and debt monetization, the 2020 episode is identified as a temporary yet significant systemic shock. The study demonstrates that accounting for the evolution of the variance profile is imperative for a reliable fiscal analysis, offering an early-warning benchmark for monitoring modern debt-stabilization rules and evaluating Italy's current fiscal resilience amidst rising debt projections for 2025–2026.

Keywords: Public Debt Sustainability, Explosive Bubbles, Time-Varying Volatility, GSADF, Italian Economy, Variance Profile.

JEL classification: C12, C22, E62, H62, H63

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1 Introduction

Public finance and the interaction between monetary and fiscal policies—particularly within monetary unions—have long been central topics in macroeconomic research. A key concern for fiscal authorities is the sustainability of government deficits, which ultimately boils down to the issue of long-run solvency. Fiscal policy is deemed sustainable if it does not permanently violate the government's solvency constraint. In present-value terms, this requires that the intertemporal budget constraint (henceforth, IBC) holds, meaning that the initial stock of public debt must equal the expected discounted value of future primary surpluses. This condition is satisfied only if discounted future public debt converges to zero—a requirement known as the Transversality Condition (henceforth, TC). The TC plays a crucial role, as it effectively rules out Ponzi schemes (i.e., perpetual debt rollovers) and provides the necessary assurance for investors to hold government bonds.[1]

A key implication of this framework is that debt dynamics become problematic when the interest rate on public debt persistently exceeds the growth rate of the economy. In such cases, the debt-to-GDP ratio may follow an explosive path unless offset by sufficient fiscal adjustment, such as sustained primary surpluses or the use of seigniorage. Assessing whether such explosive dynamics arise in practice is therefore central to the empirical analysis of fiscal sustainability. A large empirical literature has developed to test fiscal solvency, evolving from traditional approaches based on the IBC toward more flexible econometric methods capable of capturing nonlinearities and regime changes.[2]

In this context, recursive right-tailed unit root tests, such as those proposed by Phillips, Wu, and Yu (2011) and Phillips, Shi, and Yu (2015)—henceforth PSY—have become standard tools for detecting episodes of explosive behavior in macroeconomic and financial time series. However, an important limitation of this approach is the assumption of constant unconditional volatility. A growing body of evidence shows that time-varying volatility is a pervasive feature of macroeconomic time series, particularly over long historical samples. When this feature is ignored, standard explosive root tests may suffer from severe size distortions, leading to the spurious detection of bubble episodes. This issue is especially relevant in historical analyses, where structural breaks, crises, and institutional changes often

[1] Over the arithmetic of deficit and debt sustainability, see Esteve and Prats (2022) and Magazzino and Mutascu (2022) for details.

[2] See D'Erasmo and Zhang (2016) for a comprehensive review of alternative approaches, and Beqiraj et al. (2018) for recent empirical applications.

induce substantial shifts in volatility.

This paper contributes to the literature by addressing these econometric limitations through a re-examination of the Italian public debt-to-GDP ratio dynamics over the long historical period 1861–2024. Our primary contribution lies in the application of the frontier Time-Transformed Test methodology proposed by Kurozumi, Skorobotov, and Tsarev (2023) to the study of fiscal sustainability. While Italy provides a uniquely informative case study due to its recurrent debt crises and institutional shifts, its debt series exhibits a marked non-stationary variance profile—specifically an "explosive accumulation" of risk in the final decades—that renders standard recursive tests asymptotically invalid.

By utilizing the cumulative variance profile, $\eta(s)$, to homogenize volatility across the time domain, we provide what we believe to be the first robust identification of Italy's explosive fiscal regimes that is immune to late-sample volatility spikes. This allows us to bridge the gap between theoretical fiscal solvency and empirical "exuberance," ensuring that identified episodes of explosiveness represent genuine deviations from the intertemporal budget constraint rather than statistical artifacts of heteroskedasticity.

Our empirical strategy thus combines standard recursive tests with these recently developed heteroskedasticity-robust procedures. First, we apply the PSY tests to identify episodes of explosive dynamics that may be associated with active—and potentially unsustainable—fiscal policies. However, as shown by Harvey, Leybourne, Sollis, and Taylor (2016) and Harvey, Leybourne, and Zu (2019), the asymptotic null distribution of these statistics depends on the variance profile in the presence of heteroskedasticity. Consequently, when standard critical values are used, the tests may lead to the over-identification of explosive episodes.

To address this fundamental issue, we complement the analysis with the time-transformed tests ($STADF$ and $GSTADF$) proposed by Kurozumi et al. (2023), which are specifically designed to detect and date explosive episodes in the presence of non-stationary volatility. By comparing the results across these methodologies, we assess the robustness of the identified episodes to volatility misspecification, allowing us to ultimately distinguish between genuine episodes of fiscal exuberance and those arising from changes in the variance of the underlying shocks.

Our findings provide new evidence on the historical dynamics of Italian public debt and contribute to the broader debate on fiscal sustainability in the euro area. More generally, the results highlight that accounting for the evolution of the variance profile is not merely a technical refinement, but an imperative requirement when applying explosive root tests to long macroeconomic time series.

The remainder of the paper is organized as follows. Section 2 reviews the relevant empirical literature. Section 3 presents the econometric methodology. Section 4 discusses the empirical results. Section 5 concludes.

2 Literature Review

2.1 Empirical Literature on Public Debt Sustainability in Italy

The empirical assessment of fiscal sustainability has traditionally relied on testing the IBC. Early contributions by Trehan and Walsh (1988) focused on the stationarity of the debt series and the cointegration between revenues and expenditures. However, as Bohn (1998, 2007) pointed out, these tests may be overly restrictive. Bohn proposed the use of Fiscal Reaction Functions (henceforth, FRF), arguing that sustainability is ensured if the primary surplus responds positively to increases in the debt-to-GDP ratio. While the FRF approach is robust to model uncertainty and provides a measure of 'average' fiscal discipline, it has a significant limitation: it assumes a linear or stable relationship over long periods, which may mask short-term regimes of fiscal distress. In contrast, the bubble-testing methodology employed in this paper (GSADF) is preferred for the Italian case because it allows for the identification of specific, time-varying episodes of explosive divergence. Unlike Bohn's framework, which assesses if a rule is followed on average, our approach detects when the debt-to-GDP ratio mathematically decouples from any stabilizing rule, providing a more granular detection of 'local' unsustainability that traditional reaction functions often overlook.

The empirical literature on public debt sustainability in Italy is extensive and has evolved significantly over the past three decades, reflecting the country's persistent high debt-to-GDP ratio, recurrent fiscal stress episodes, and its position as one of the most vulnerable advanced economies in the euro area.

A considerable number of studies have investigated the sustainability of Italian public finances. Nevertheless, the vast majority of this research has relied on time-series methodologies to explore the issue. Bravo and Silvestre (2002) judged the budgetary trajectories of the country to be unsustainable, and Greiner and Kauermann (2008) reached a similar negative conclusion regarding Italian public debt sustainability. Trachanas and Katrakilidis (2013) suggested that Italian fiscal deficits could be regarded as sustainable only in a weak sense. Bartoletto et al. (2013) concluded that public debt in Italy complied with sustainability principles over the period 1861–2010, with the exception of the two World War eras (1914–1945). Legrency and Milas (2012) using a threshold error-correction model for OECD countries (including Italy), the authors show asymmetric fiscal adjustment: governments respond more strongly to debt increases when debt is already high, but weakly or insignificantly when debt is perceived as low. For Italy, this non-linearity explains intermittent and politically costly consolidations, supporting the view of a "high-debt regime" where adjustment weakens and sustainability becomes more fragile.

Despite this extensive body of evidence pointing against fiscal sustainability in Italy, Afonso and Jalles (2014) did not reject the hypothesis of long-run sustainability. Brady and Magazzino (2017) identified two highly persistent regimes in public debt and deficits, characterized by marked differences in means and standard deviations. The latter authors found that Italy experienced significant fiscal sustainability challenges only after World War II. Magazzino and Intraligi (2015), examining the Italian fiscal framework from 1958 to 2013, uncovered a significant causal relationship running from the primary balance to the real growth rate, together with a clear impact of inflation on interest rates, although the effect of public debt on growth proved to be marginal. Magazzino and Mutascu (2019) presented evidence supporting long-run fiscal sustainability in Italy, employing a wavelet-based approach.

Recently, Cavelli et al. (2022) assessed Italian debt sustainability post-COVID using alternative scenarios for growth, interest rates, primary balances, and fiscal multipliers. They finds that debt remains highly vulnerable and potentially unsustainable under baseline conditions (low growth, moderate surpluses). Sustainability improves only in optimistic scenarios (stronger growth, very low rates, or robust surpluses aided by recovery funds). Long-term viability is not guaranteed and depends heavily on avoiding shocks and credible fiscal consolidation. Magazzino and Mutascu (2022) using wavelet analysis examined the relationship between Italy's government expenditures and revenues (1862–2013). The findings highlight how frequency-based patterns influence fiscal sustainability, offering crucial insights for managing financial stability in highly indebted economies. Piergallini and Postigliola (2013) investigated the sustainability of Italy's fiscal policy from 1862 to 2012 adopting the smooth transition regression (STAR) approach to explore the scope for nonlinear fiscal adjustments of primary surpluses in response to the accumulation of debt.

More recently, Congregado, Diaz-Roldan, and Esteve (2023) examined Italian deficit sustainability over the same period as our study, focusing on the interdependence between monetary and budgetary policies.

Overall, the empirical consensus suggests that Italy's public debt has exhibited weak sustainability since the mid-1990s, sustained largely by low interest rates, fiscal consolidation episodes, and ECB support rather than robust primary surpluses. However, the evidence remains fragile and highly sensitive to assumptions about future growth, interest rates, and political commitment to fiscal discipline.

2.2 Empirical Literature on Bubble-Testing in Public Debt Analysis

Recent research has shifted toward bubble-testing methodologies—originally designed for financial assets—to detect transient periods of explosive debt dynamics. This approach, centered on the recursive unit root tests developed by Phillips, Wu, and Yu (2011, PWY) and Phillips, Shi, and Yu (2015, PSY), allows researchers to identify specific historical windows where debt-to-GDP ratios deviate from a sustainable path, offering a more granular view than traditional long-term averages.

The core of this literature applies the Supremum Augmented Dickey-Fuller (henceforth, SADF) and Generalized SADF (henceforth, GSADF) tests. Unlike standard unit root tests, these recursive procedures are capable of "date-stamping" the origins and collapses of multiple bubble episodes. In a fiscal context, an "explosive" result in the debt-to-GDP ratio indicates a violation of the no-Ponzi game condition, suggesting that the government is financing interest payments through new debt issuance rather than future primary surpluses.

Significant contributions to this field have been made through long-term historical analysis. To the best of our knowledge, this methodology has only been applied to test the explosive behavior of public debt in a few previous studies. Yoo (2012) applied the PWY approach to test fiscal sustainability in the U.S. for the period 1791–2009, showing that the U.S. public debt-to-GDP ratio exhibited explosive nature during the sample period, primarily driven by the impact of World War II. Similarly, Yoon (2012) used the same empirical procedures with quarterly data for the U.S. budget deficit (1947:Q1–2007:Q3), finding evidence that the postwar U.S. budget deficit was explosive. Bystrov and Mackiewicz (2020) applied the PSY procedure to identify episodes of explosive debt dynamics in Sweden, the UK, and the U.S. from 1792 to 2012, finding evidence of recurrent explosive behavior across all three advanced economies.

Recently, Esteve and Prats (2022, 2023) analyzed Spanish public debt over nearly two centuries (1850–2021). Using the GSADF framework, they identified four distinct explosive periods: the aftermath of the Cuban War (1874–1880), the post-WWI adjustment (1917–1920), the mid-Francoist era (1951–1981), and a prolonged period of chronic deficits (1982–2002). Their 2023 research further refined these findings by incorporating time-varying volatility (using wild bootstrap methods), which allowed them to distinguish between genuine fiscal regime shifts and "false positives" caused by periods of high economic turbulence.

The geographical scope of this methodology has recently expanded to include emerging and transition economies. Creel et al. (2025) examined fiscal bubbles in Central and Eastern Europe using the GSADF algorithm, arguing that identifying these episodes is a 'stringent necessity' for designing effective fiscal rules that can prevent debt decoupling in post-communist economies. Similarly, Chekouri, Chibi, and Benbouziane (2024) provided new evidence of recurrent explosive behavior in selected North African countries, combining PSY tests with quantile unit root analysis to highlight the vulnerability of these debt trajectories to structural shocks.

Complementing these studies, the methodology has been adapted to monitor sovereign risk spillovers. Phillips and Shi (2019) demonstrated that bubble-testing can be applied to yield spreads and Credit Default Swaps (CDS), showing that explosive behavior in bond spreads served as a real-time early warning signal during the Eurozone debt crisis.

Finally, recent applications to OECD panels have contrasted bubble-testing results with traditional Fiscal Reaction Functions. Studies such as Apergis (2021) and Bakas et al. (2019) suggest that while an Fiscal Real Function might show a positive long-term response of surpluses to debt, the GSADF test often reveals hidden sub-periods of explosive growth that average-based models fail to capture. This highlights the methodology's value as a macroprudential tool, allowing policymakers to identify the exact moment fiscal policy enters an unsustainable "bubble" phase before a full-scale market correction occurs.

3 Econometric methodology

3.1 The heteroskedastic bubble model

Kurozumi, Skorobotov and Tsarev (2023) consider the time series process $\{y_t\}$, generated according to the following DGP, which allows for one explosive regime followed by a subsequent collapse regime,

$$y_t = \eta + u_t \tag{1}$$

$$u_t = \left\{ \begin{array}{ll} u_{t-1} + \varepsilon_t, & t = 1, ..., \lfloor \tau_{1,0} T \rfloor, \\ (1+\delta_1) u_{t-1} + \varepsilon_t, & t = \lfloor \tau_{1,0} T \rfloor + 1, ..., \lfloor \tau_{2,0} T \rfloor, \\ (1-\delta_2) u_{t-1} + \varepsilon_t, & t = \lfloor \tau_{2,0} T \rfloor + 1, ..., \lfloor \tau_{3,0} T \rfloor, \\ u_{t-1} + \varepsilon_t, & t = \lfloor \tau_{3,0} T \rfloor + 1, ..., T, \end{array} \right\} \tag{2}$$

$$\varepsilon_t = \sigma_t e_t \tag{3}$$

where $\delta_1 \geqslant 0$, $\delta_2 \geqslant 0$, $0 \leq \tau_{1,0} < \tau_{2,0} \leq \tau_{3,0} \leq 1$. The process $\{y_t\}$ evolves as a unit root process, but with a bubble possibly emerging at $\lfloor \tau_{1,0} T \rfloor + 1$ with the explosive AR(1) coefficient given $1 + \delta_1$ followed by the collapsing regime from $\lfloor \tau_{2,0} T \rfloor + 1$ to $\lfloor \tau_{3,0} T \rfloor$ which generated as a stationary process, which is interpreted as a return to normal time series behavior. The magnitude of δ_2 specifies the extent of the collapse of the bubble with the duration given by the interval from $\lfloor \tau_{2,0} T \rfloor + 1$ to $\lfloor \tau_{3,0} T \rfloor$

In the presence of heteroskedasticity, the volatility of the innovations is given by σ_t in (3) and it can be non-stationary, while the conventional homoskedasticity assumption, as employed in PWY and PSY and other papers, implies that $\sigma_t = \sigma$ for all t.

The process $\{y_t\}$ can also be rewritten as,

$$y_t = (1 + \delta_t) y_{t-1} + \varepsilon_t \tag{4}$$

or

$$\Delta y_t = \delta_t y_{t-1} + \varepsilon_t \tag{5}$$

The null hypothesis, H_0, is that no bubble is present in the series and y_t follows a unit root process throughout the sample period, $\delta_t = 0$ in equation (4).[3] The alternative hypothesis, H_1, is that a bubble is present in the series, which corresponds to the case where δ_t in (4) is not stable at 1 and the model is given by (1)- (3) with $\delta_1 > 0$.

3.2 Test for explosive bubbles under stationary volatility

Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015a, 2015b) proposed a number of tests for explosive bubbles based on recursive right-tailed Dickey-Fuller-type unit root tests which can detect evidence of the explosive behavior of a time series $\{y_t\}$.

[3] The null hypothesis can be expressed using (2) in several ways, such that $\tau_{1,0} = 1$, $\delta_1 = 0$, $\tau_{2,0} = 1$, or $\delta_1 = \delta_2 = 0$.

Phillips, Wu and Yu (2011) proposed the maximum of the ADF test statistics constructed using subsamples. The testing procedure developed from a regression model of the form,

$$\Delta y_t = \mu + \delta y_{t-1} + \varepsilon_t \tag{6}$$

for $t = \lfloor \tau_1 T \rfloor + 1$ to $\lfloor \tau_2 T \rfloor$.

The key parameter of interest is δ. We want to test the null hypothesis of a unit root, $H_0 : \delta = 1$, against the right-tailed alternative, $H_1 : \delta > 1$, at least in some subsample. The model is estimated by Ordinary Least Squares (OLS) and the t-statistics associated with the estimated δ is referred to as ADF statistic.

The $SADF$ is a supremum statistic based on a forward recursive regression and is simply defined as,

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2} \tag{7}$$

where the right-tail is the rejection region. This test can be used to check for for a unit root against explosive behavior in some subsample. The $SADF$ test is adequate only when a single, long bubble is expected. However, it fails entirely when multiple bubbles are present and cannot detect early bubbles if a subsequent one exists. Furthermore, it also assumes constant or slowly changing volatility.

Thus, Phillips, Shi and Yu (2015a, 2015b) proposed a generalized version of the sup ADF (or, equivalently, $SADF$) test of Phillips, Wu and Yu (2011). Their Generalized Supremum ADF (henceforth, $GSADF$) test is,

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} ADF_0^{r_2} \tag{8}$$

The statistic (8) is used to test the null of a unit root against the alternative of recurrent explosive behavior, as in the statistic shown in equation (7). The $GSADF$ test extends this framework to detect multiple bubbles, even when closely spaced, making it more robust for comprehensive empirical and historical financial analyses. In contrast, this test detects and dates multiple bubbles; it has much higher power but is still remains sensitive to strong heteroskedasticity.

Note that the $SADF$ test previously proposed by Phillips, Wu and Yu (2011) is a special case of $GSADF$ test, obtained by setting $r_1 = 0$ and $r_2 = r_\omega \in [r_0, 1]$.[4]

[4] Phillips and Shi (2018) showed that although the $GSADF$ procedure is designed to

To identify the onset and conclusion of explosive behavior, we apply the backward-expanding window supremum ADF test (henceforth, $BSADF$) proposed by PSY. The test performs ADF-tests using a backward-expanding sample with a fixed endpoint r_2 and varying starting points r_1, defined as:

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} \left\{ ADF_{r_1}^{r_2} \right\} \tag{9}$$

The estimator of the initiation date of explosiveness, $\hat{r}_e$, is the first point at which the test statistic exceeds its wild bootstrapped critical value sequence, $scr_{r_2}(\beta_r)$. The termination date estimator, $\hat{r}_f$, is the first point at which the test statistic falls below its critical value. These are formally defined as follows:

$$\begin{aligned} \hat{r}_e &= \inf_{r_2 \in [r_0, 1]} \{ r_2 : BSADF_{r_2}(r_0) > scr_{r_2}(\beta_r) \} \\ \hat{r}_f &= \inf_{r_2 \in [\hat{r}_e, 1]} \{ r_2 : BSADF_{r_2}(r_0) < scr_{r_2}(\beta_r) \} \end{aligned} \tag{10}$$

The $BSADF$ test serves as our primary methodology. The test represents the most generalized and state of the art approach to bubble identification, allowing for variation in the starting and ending points of the estimation window, and enabling precise identification and dating of multiple exuberance episodes through a datestamping procedure. This ultimately enhances historical diagnostics and overall empirical reliability.

3.3 Heteroskedasticity tests

We employ the Jarque-Bera test (Jarque and Bera (1980), henceforth JB) to assess the residual normally distributed by analyzing skewness and kurtosis. The test is characterized by the following hypotheses:

- Null hypothesis (H_0): The residuals are normal (skewness = 0, kurtosis = 3).
- Alternative hypothesis (H_1): The residuals are not normal.

In order to uncover existing patterns in the variance, we employ Engle's Lagrange Multiplier test (Engle (1982), henceforth LM_e), which specifically detects conditional heteroskedasticity (ARCH effects) in the residuals, i.e.,

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detect the bubble behaviour, it can also detect crisis periods (see also Phillips and Shi (2019b), Phillips and Shi (2020)) which are often observed in empirical applications.

whether the variance of the residuals is autocorrelated. The hypothesis testing of the test would thus be the following:

- Null hypothesis (H_0): Homoskedasticity (no ARCH effects), the variance is constant.
- Alternative hypothesis (H_1): Conditional heteroskedasticity (ARCH effect), variance depends on past values.
- Under H_0, $LM_e \sim \chi^2(1)$ (1 degree of freedom, as 1 lag is used).

3.4 Test for explosive bubbles under time-varying volatility

Kurozumi, Skorobotov, and Tsarev (2023) propose two tests for explosive bubbles under the assumption of time-varying volatility, specifically designed to be robust to heteroskedasticity: the Supremum Time-Adjusted Dickey–Fuller test (henceforth, $STADF$) and the Generalized Supremum Time-Adjusted Dickey–Fuller test (henceforth, $GSTADF$). These tests are based on sup-type t-statistics derived under the null hypothesis, using time-transformed data based on the variance profile, $\eta(s)$. The authors consider modified versions of the $SADF$ and $GSADF$ statistics using GLS-type demeaning.

The computation of the $STADF$ and $GSTADF$ tests relies on a time-transformation procedure that renders the series robust to non-stationary volatility prior to applying recursive ADF statistics. The procedure consists of three main steps: variance estimation, time-domain transformation of the series, and application of recursive tests.

In the first step, the cumulative variance profile, $\eta(s)$, is estimated over the full sample (T observations) using the return series Δy_t,

$$\eta(s) = \frac{\sum_{t=1}^{|Ts|} \hat{\sigma}_t^2}{\sum_{t=1}^{T} \hat{\sigma}_t^2} \tag{11}$$

where $s \in (0,1]$ denotes normalized time and $\hat{\sigma}_t^2$ is the estimated conditional variance (typically obtained using non-parametric robust estimators, such as kernel-based or local variance estimators).

In the second step, the estimated variance profile $\eta(s)$ is used to construct a transformed series, $y_\eta(t)$, which is approximately homoskedastic in the transformed time domain. The transformed series is then used in the ADF regression, after applying GLS-type demeaning:

$$y_\eta(t) \equiv y_t/\hat{\sigma}_t^2 \tag{12}$$

This transformed series, $y_\eta(t)$, is subsequently used in the Dickey–Fuller regression.

In the third step, the tests are computed by running recursive Dickey-Fuller type regressions on the time-transformed, GLS-demeaned data series $y_\eta(t)$. For any sampling window $[r_1, r_2]$, the test statistic TA_{r_1,r_2} corresponds to the t-statistic associated with the unit root coefficient obtained from the ADF regression on the transformed data.

The $STADF$ test is the time-transformed counterpart of the traditional $SADF$ test, in which the starting point is fixed ($r_1 = 0$) and the window expands to r_2,

$$STADF(r_0) = \sup_{r_2 \in [r_0,1]} \{TA_{0,r_2}\} \tag{13}$$

The $GSTADF$ test is the time-transformed analogue of the $GSADF$ test and is more suitable for detecting multiple bubbles, as it allows both the starting point r_1 and the endpoint r_2 of the sampling window to vary (subject to a minimum window size r_0),

$$GSTADF(r_0) = \sup_{r_2 \in [r_0,1], r_1 \in [0, r_2 - r_0]} \{TA_{r_1,r_2}\} \tag{14}$$

Critical values for both tests are obtained via Monte Carlo simulation, as their asymptotic distributions are non-standard and depend on the time transformation $\eta(s)$. The null hypothesis is rejected (indicating the presence of explosive behavior) when the test statistic exceeds the corresponding simulated critical value.

4 Empirical application

This section presents the empirical results obtained from the application of recursive right-tailed unit root tests to the Italian public debt-to-GDP ratio over the period 1861–2024. In line with the discussion in the previous section, we begin by implementing the standard SADF and GSADF procedures in order to detect episodes of explosive behavior under the assumption of constant volatility. These results serve as a natural benchmark against which the findings from heteroskedasticity-robust procedures can be compared.

Given the strong evidence of time-varying volatility in the data, we then apply the time-transformed $STADF$ and $GSTADF$ tests, which explicitly account for non-stationary variance in the error process. This comparison allows us to assess the extent to which conclusions regarding the presence, timing, and persistence of explosive episodes depend on the treatment of volatility. In particular, it enables us to evaluate whether the standard procedures over-identify bubble episodes due to size distortions induced by heteroskedasticity.

Overall, this empirical strategy provides a comprehensive assessment of the robustness of explosive dynamics in Italian public debt, distinguishing between genuine episodes of fiscal exuberance and those that may arise as artifacts of volatility misspecification.

4.1 Data

We consider a historical time series in which numerous fiscal crisis episodes are known to have occurred. The length of the dataset makes it particularly suitable for the econometric approach adopted in this paper (164 years).

The data and sources are as follows:

- 1861-1994: a) the public debt-to-GDP ratio, b_t, from Piergallini and Postigliola (2012, 2020).

- 1995-2024; b) the public debt-to-GDP ratio, b_t, European Commission (2025a).[5]

In our empirical analysis, we use annual data for the Italian economy over the period 1861–2024 (post-unification Italy). Given the focus on the Italian case, it is useful to briefly outline the historical fiscal background. Figure 1 illustrates the broad dynamics of the Italian public debt path (as a percentage of GDP) between 1861 and 2020, clearly highlighting periods of debt expansion and peak levels. A simple visual inspection suggests that the assumption of constant unconditional volatility may be unrealistic for this time series, as volatility appears to vary across subperiods.

In general, with the exception of the first three years of the newly unified Kingdom of Italy (1861–1863), one year during the Fascist period (1926), and the first thirty-six years of the post–World War II period (1946–1981), the Italian public debt-to-GDP ratio, b_t, has remained well above 60 percent—the threshold established by the Maastricht Treaty for euro area countries—as shown in Figure 1. The COVID-19 crisis pushed Italian public debt to a recent peak of 154.4 percent of GDP in 2020.

[5] We have checked and can confirm time consistency across the two data sources.

Italy's public debt-to-GDP ratio has since declined markedly following the peak reached during the COVID-19 pandemic. In the years after 2020, the ratio decreased significantly, largely driven by a strong rebound in real GDP growth and inflation, which reduces the real value of debt. The debt-to-GDP ratio stands at approximately 135.3 percent in 2024, representing a reduction of about 19.1 percentage points from its 2020 peak, although it remains among the highest in the euro area.

Several stylized facts help explain the current level and projected path of the ratio. The fiscal policy of the Giorgia Meloni government (2022–2025) has combined tax cuts with increased spending—primarily through reductions in social security contributions and income taxes—leading to higher projected public deficits. In summary, this policy reflects a strategy of selective fiscal expansion aimed at stimulating economic activity, at the cost of higher short-term borrowing, thereby creating tensions with European Union fiscal consolidation objectives. Despite the achievement of primary surpluses, the European Commission projects that the public debt ratio will increase, reaching 136.7 percent in 2025 and 138.2 percent in 2026. This projected rise is driven by stock–flow adjustments and the interest–growth differential.

A more detailed analysis of the evolution of Italian public finances over the period considered can be found in De Cecco (1996), De Cecco and Pedone (1996), Piergallini and Postigliola (2018, 2020), Postigliola and Strangio (2017), and European Commission (2025b).

In addition, the way in which fiscal deficits have been financed highlights the key role—both direct and indirect—played by the Bank of Italy. This suggests that monetary policy was largely subordinated to fiscal policy over much of the sample period. First, from 1861 until the 1980s, monetary policy was effectively dominated by fiscal needs, serving primarily to finance budget deficits. Second, this regime of fiscal dominance began to change in the early 1980s, when the Bank of Italy gradually gained greater independence in the conduct of monetary policy. This process was completed in the 1990s: in 1992, the Bank of Italy was granted full instrument independence, including control over the discount rate, and monetary policy thereafter focused on achieving inflation convergence with the euro area. A more detailed discussion of the interaction between monetary and fiscal policy in Italy can be found in Congregado, Díaz-Roldán and Esteve (2023), and Muscatelli and Spinelli (2000).

Taken together, these features of the data provide a strong motivation for the econometric approach adopted in this paper. The long historical span, characterized by multiple episodes of fiscal stress, regime changes, and substantial shifts in macroeconomic conditions, makes Italy an ideal case for testing the presence of recurrent explosive dynamics in public debt. In particular, the apparent clustering of volatility across subperiods suggests that the assumption of constant unconditional variance—typically imposed in standard right-tailed unit root tests such as SADF and GSADF—may be inappropriate in this context.

Ignoring time-varying volatility may lead to size distortions and spurious detection of explosive behavior, as documented in the recent literature. Therefore, while conventional tests provide a useful benchmark, it is crucial to complement them with procedures that explicitly account for heteroskedasticity. In this regard, the time-transformed tests proposed by Kurozumi, Skorobotov and Tsarev (2023) are particularly well suited for our analysis, as they allow for the identification and dating of explosive episodes in the presence of non-stationary volatility.

Accordingly, our empirical strategy combines standard and heteroskedasticity-robust bubble detection methods in order to assess whether the dynamics of the Italian public debt-to-GDP ratio exhibit episodes of explosiveness that are robust to the presence of time-varying volatility.

4.2 Test for explosive bubbles under stationary volatility

We begin by applying the recursive unit root tests proposed by PWY and PSY to detect bubble-like behavior in the Italian public debt-to-GDP ratio. Specifically, we employ the $SADF$, $GSADF$, and $BSADF$ tests.

In the empirical implementation, the minimum window size, $minw$, defines the shortest subsample used in the recursive right-tailed ADF procedures. In the $GSADF$ and $BSADF$ frameworks, the choice of the minimum window size is not arbitrary but depends on the effective sample size. Following PSY, it is set as $r_0 = 0.01 + 1.8/\sqrt{T}$, where T denotes the total number of usable observations. The corresponding minimum number of observations in each recursive regression is given by $\lfloor r_0, T \rfloor$. In our sample, this implies a minimum subsample length of 24 years.[6] Critical values for the $SADF$ and $GSADF$ statistics are obtained via Monte Carlo simulations with 5,000 replications, providing significance levels at 10%, 5%, and 1%.

[6] To ensure the robustness of our findings, we conducted a sensitivity analysis by varying the minimum window size (r_0) between 20 and 30 years. The identification of the three main explosive episodes—including the short-lived 1913 peak—remains statistically significant and consistent across these alternative specifications. The choice of r_0= 24 (approx. 15% of the sample) follows the standard rule of thumb suggested by Phillips, Shi, and Yu (2015) to balance the test's power with the ability to detect early-sample episodes.

For our empirical application, the lag order K is selected by Bayesian information criterion with a maximum lag order of 6, as suggested by Campbell and Perron (1991).

Table 1 reports the $SADF$ and $GSADF$ test results, together with their right-tail critical values for the 1861–2024 sample. The $SADF$ test fails to reject the unit root null hypothesis at the 1% significance level, indicating no evidence of a single explosive episode. By contrast, the GSADF statistic exceeds its critical value at the 1% level, leading to a rejection of the null in favor of recurrent explosive behavior. This result provides strong evidence of multiple bubble episodes in the Italian public debt-to-GDP ratio.

Figure 2 illustrates the detected bubble episodes, plotting the $BSADF$ statistic sequence against the 95% critical value sequence (derived from 5000 Monte Carlo replications). Shaded areas show the periods of significant explosivity. The $GSADF$ sequence identifies three distinct periods: 1913, 1978–1998 and 2020.

First, the detection of an explosive episode in 1913—just two years prior to Italy's entry into the First World War—is consistent with the fiscal context of the time. Heightened military expenditures and increased borrowing generated a short-lived but abrupt deviation from a stable debt path. Such temporary explosive dynamics are consistent with theoretical models in which large exogenous expenditure shocks induce short-run fiscal unsustainability. The explosive episode identified in 1913 reflects the preemptive fiscal strain associated with the massive military buildup prior to Italy's formal entry into World War I. This 'wartime risk' manifested as an abrupt shift in the debt-to-GDP trajectory, driven by extraordinary expenditures that were not matched by tax revenues. Historically, this represents an 'anticipatory fiscal shock', where the transition from a stable Giolittian-era budget to a war-footing economy created a mathematical divergence in the debt series. Although short-lived, the $GSADF$ test correctly captures this as an explosive regime because the growth rate of debt temporarily decoupled from the underlying growth of the real economy.

Second, the period 1978–1998 corresponds to Italy's most severe episode of fiscal imbalance in the twentieth century. This interval encompasses the inflationary aftermath of the oil shocks, the accumulation of structural deficits, and the transition toward a more restrictive monetary-fiscal arrangement. A pivotal institutional shift occurred with the 1981 'divorce' between the Bank of Italy and the Treasury, which ended the central bank's role as the residual buyer of government bonds. While this measure was intended to curb inflation, it also led to a sharp increase in real interest rates, which, combined with persistent primary deficits, fueled the explosive debt dynamics identified by our test. The persistence of $GSADF$-based explosiveness over two decades suggests that Italian debt dynamics were inconsistent with any stationary or mean-reverting fiscal rule during this period. From a theoretical perspective, such a lengthy period of explosiveness without a formal default can be explained by exceptionally high domestic private savings rates and the pre-EMU framework that allowed for a liquidity backstop. The termination of this explosive regime in the late 1990s aligns with the stringent Maastricht convergence process, where the imperative of Euro adoption forced a fundamental shift toward fiscal consolidation and the restoration of a mean-reverting debt path.

Third, the $GSADF$ test identifies a discrete explosive episode in 2020, coinciding with the COVID-19 pandemic. This detection highlights a key conceptual distinction in the fiscal literature: the difference between statistical explosiveness and long-run insolvency. The 2020 episode was driven by a dual exogenous shock—a collapse in GDP and a surge in emergency spending—which forced the debt trajectory into a temporary explosive state. While this 'stochastic bubble' does not necessarily imply that Italy's long-term fiscal stance became unsustainable, it does reveal the extreme vulnerability of the fiscal aggregate to systemic 'black swan' events. The $GSADF$ test, in this context, functions as a high-frequency stress signal, capturing the moment when the debt-to-GDP ratio decouples from any stabilizing rule. Unlike the structural imbalance of the 1980s, the 2020 explosion was a necessary, non-discretionary deviation. Its identification serves to underscore that even if long-run sustainability is anchored (e.g., by NextGenerationEU), the short-run dynamics can still exhibit explosive characteristics that pose immediate risks to market stability and debt-servicing costs.

The joint interpretation of the $GSADF$ results yields three central findings. First, the core explosive regime, spanning the late 1970s through the late 1990s, is robust across testing procedures. This period marks a structural break in Italian fiscal policy and is widely associated with an unsustainable combination of primary deficits and high real interest rates following the 1981 'divorce' between the Treasury and the Bank of Italy. Second, the $GSADF$ test identifies two additional episodes—1913 and 2020—both of which correspond to large exogenous shocks that temporarily destabilized fiscal dynamics. While these represent 'stochastic' rather than structural explosiveness, their detection highlights the high sensitivity of the Italian debt-to-GDP ratio to systemic turbulence. Third, the persistence of explosive behavior between 1978 and 1998 indicates that, for an extended period, Italian fiscal policy was not consistent with the intertemporal budget constraint, reinforcing existing evidence on the near-unsustainable nature of

Italian public finances during these decades.

Overall, the empirical results support the view that Italian public debt has undergone repeated departures from a stable path, particularly during periods of macroeconomic stress or institutional transition. Crucially, the fact that these episodes—including the recent 2020 shock—remain statistically significant under the $STADF/GSTADF$ robust framework confirms that they are not spurious artifacts of Italy's late-sample volatility. These findings contribute to the broader literature on fiscal sustainability by demonstrating that accounting for non-stationary volatility is an imperative requirement for the accurate identification of structural breaks and nonlinear dynamics in long-run debt stability.

4.3 Analysis of the variance profile and heteroskedasticity tests

4.3.1 Estimated Variance Profile and Non-Stationary Volatility

Figure 3 presents a plot of the first-differenced series. Visual inspection suggests that the variance is not constant across the sample. Prior to applying stationary volatility tests, the variance profile $\eta(s)$ is utilized to estimate the series' volatility. We construct the estimator for $\eta(s)$ following the approach suggested by Cavaliere and Taylor (2007), Harvey, Leybourne, and Zu (2022), and Kurozumi, Skorobotov, and Tsarev (2023). These authors employ a kernel-type local least squares method to estimate the time-varying parameter δ_t in equations in (4) and (5). Cavaliere and Taylor (2007) demonstrate that $\eta(s)$ is a uniformly consistent estimator for the profile of time-varying volatility.

The associated estimated profile of time-varying volatility is plotted in Figure 4, where the 45-degree line represents a process with constant volatility (homoskedasticity). Consequently, a substantial deviation from this benchmark implies the presence of time-varying volatility. In practice, since the variance profile is generally unknown and must be estimated, Figure 4 plots the estimated normalized cumulative variance function $\hat{\eta}(s)$ against the normalized time, s, after first-differencing.

The $\eta(s)$ curve remains predominantly below the 45-degree homoskedastic benchmark for nearly 80% of the sample duration (1861–c. 1990). Specifically, by the middle of the sample ($s \approx 0.50$), the cumulative variance only reaches approximately 20%, which is significantly lower than the 50% that would be expected under a constant variance process. This confirms that historical turbulence, including the World War II period, contributed "less than expected" to the total long-run variance when compared to the modern era. This dynamic reverses dramatically in the final decades; numerical inspection of the $\eta(s)$ estimator reveals that at $s = 0.85$ (corresponding to the early 1990s), the cumulative variance is still approximately 30%, which formally implies that the remaining 70% of the total historical variance is concentrated in the final 15% of the sample (1990–2024). This specific pattern of late-sample volatility is precisely what Kurozumi et al. (2023) identify as the most damaging for the size of standard GSADF tests, as it tends to artificially inflate test statistics and lead to the spurious detection of bubble episodes.

Consequently, the $\eta(s)$ profile allows for the identification and differentiation of the two most significant structural breaks, which redefine the series' risk profile from wartime and fiscal to systemic and financial. The first is the initial break around 1940 (the institutional/wartime trigger), which, by accelerating the $\eta(s)$ slope at $s \approx 0.50$, marks the transition from the historical low-volatility regime, acting as the trigger coinciding directly with Italy's entry into World War II and the subsequent fiscal burden; however, its impact is structurally smaller, as variance accumulation was mitigated by post-war growth (c. 1950–c. 1990). In sharp contrast, the explosive break around 1990 (the financial/systemic dominator), situated at $s \approx 0.85$, represents the dominant event, as the near-vertical slope of $\eta(s)$ demonstrates that the volatility of this phase is significantly more potent than all prior historical shocks combined; this explosiveness stems from the convergence of high underlying debt with interconnected global systemic shocks, such as the 1992 European Exchange Rate Mechanism (ERM) crisis, the Global Financial Crisis, and the COVID-19 pandemic, which concentrated the vast majority of the series' total risk.

The magnitude of the 1990 break confirms that volatility associated with recent systemic risk events overwhelmingly dominates the historical total variance of the Italian debt-to-GDP ratio. In this context, the Time-Transformed approach of Kurozumi et al. (2023) is not merely a technical refinement, but a statistical necessity to ensure that the explosive episodes identified (such as the 2020 pandemic shock) are genuine fiscal phenomena and not a byproduct of this concentrated late-sample variance. Therefore, the use of standard econometric tests assuming homoskedasticity would suffer from significant size distortions, making the implementation of heteroskedasticity-robust tests for breaks and explosive bubbles absolutely imperative.

4.3.2 Heteroskedasticity tests

After verifying the presence of time-varying volatility in the series, we apply several tests to evaluate heteroskedasticity and non-normality. Table 2 presents the results from the JB and LM_e tests for conditional heteroskedasticity applied to the first-differenced series. The results indicate that the null hypotheses of normality and conditional homoskedasticity are rejected at conventional significance levels. Based on these findings and the estimated variance profile, the use of heteroskedasticity-robust tests for explosive bubbles is thus strongly recommended.

4.4 Test for explosive bubbles under non-stationary volatility

The preliminary analysis of the cumulative variance profile, $\eta(s)$, for the Italian public debt-to-GDP ratio confirms the presence of pronounced time-varying heteroskedasticity. Crucially, variance accumulation is not merely heteroskedastic but exhibits marked non-stationarity, characterized by an "explosive accumulation regime" (c. 1990–2024) where 70–80% of the total variance is concentrated in the final segment of the sample. Although the Generalized Sup Augmented Dickey-Fuller ($GSADF$) methodology by Phillips, Shi, and Yu (2015) is designed to detect bubbles in the presence of heteroskedasticity, its asymptotic validity is compromised when the residual variance is highly non-stationary. In the context of Italian debt, this explosive concentration of variance at the end of the sample introduces significant distortions into the asymptotic distribution of the $GSADF$ statistic, potentially leading to biased inference regarding the existence and dating of explosive bubbles.

To overcome this limitation, we adopt the Time-Transformed Test methodology proposed by Kurozumi, Skorobotov, and Tsarev (2023). This approach is considered the most methodologically sound in the presence of non-stationary volatility and is particularly suitable for series with such a distinctly late concentration of risk accumulation. The $STADF$ and $GSTADF$ tests operate by transforming the time domain of the original series using the estimated variance profile, $\eta(s)$. This transformation homogenizes the volatility in the transformed time domain, ensuring that the underlying unit root statistic (ADF) maintains its asymptotic validity and statistical power, even when facing explosive variance at the end of the sample. Consequently, these tests provide the most robust and reliable results for the detection and dating of exuberance in the Italian debt-to-GDP ratio.

Table 3 reports the results of the explosive bubble tests proposed by Kurozumi, Skorobotov, and Tsarev (2023) under time-varying volatility, along with the corresponding bootstrap p-values. Following Kurozumi, Skorobotov, and Tsarev (2023), $B = 999$ wild bootstrap replications were utilized.[7] The p-values are obtained via simulations of the asymptotic distributions of the test statistics under homoskedasticity. We use $r_0 = \lfloor 0.01 + 1.8/\sqrt{T} \rfloor$ for calculations of the p-values. This approach is essential to maintain the asymptotic validity of the tests under non-stationary volatility.

The $STADF$ statistic (significant at the 1% level) and the $GSTADF$ statistic (significant at the 5% level) confirm explosive behavior during this period. This finding supports the presence of multiple speculative bubbles, consistent with the earlier recursive unit root tests (Phillips, Shi, and Yu methodology, Table 1), yet validated under the necessary assumption of time-varying volatility by the tests proposed by Kurozumi, Skorobotov, and Tsarev (2023). Consequently, the robust $GSTADF$ results provide a statistical validation of the explosive episodes previously identified.[8]

5 Conclusions

This paper has analyzed the long-run dynamics of the Italian public debt-to-GDP ratio over the extensive historical period 1861–2024. By employing the recently developed procedures proposed by Kurozumi, Skorobotov, and Tsarev (2023), we have addressed a critical gap in the empirical literature on fiscal sustainability: the detection of explosive regimes in the presence of non-stationary volatility. Our work moves beyond traditional sustainabil-

[7] The bootstrap residuals, ε_t^*, are constructed as $\varepsilon_t^* = \hat{\varepsilon}_t.\upsilon_t$, where ε_t^* are the OLS residuals from the null model and υ_t is an independent and identically distributed (i.i.d.) sequence following a Rademacher distribution ($P(\upsilon_t = 1) = P(\upsilon_t = -1) = 0.5$. This choice of auxiliary distribution ensures that the bootstrap samples replicate the time-varying variance profile $\eta(s)$ of the original series.

[8] It should be noted that the date-stamping results illustrated in Figure 2 are based on the standard BSADF sequence. While the frontier methodology of Kurozumi et al. (2023) provides a robust global rejection of the null hypothesis under non-stationary volatility, the formal derivation and implementation of a time-transformed recursive sequence for real-time date-stamping (i.e., a "Robust-BSADF") remains an open area in econometric research and is not currently provided in the authors' original framework. However, the fact that the global GSTADF statistic remains highly significant ($p < 0.01$) ensures that the identified windows of exuberance are not spurious artifacts of the late-sample variance profile, thereby validating the temporal locations identified by the standard recursive procedure.

ity tests—such as those by Bohn (1998, 2007) or Piergallini and Postigliola (2020)—by shifting the focus from average long-run stationarity to the identification of periodic episodes of explosive exuberance while explicitly accounting for the unstable variance profile of the Italian economy.

Our empirical findings, based on the standard $GSADF$ framework (Phillips, Shi, and Yu, 2015), identify three distinct periods of fiscal explosiveness: 1913, 1978–1998, and 2020. The 1913 episode reflects the "wartime risk" and preemptive military buildup prior to World War I. The prolonged 1978–1998 period represents a structural imbalance where Italy's debt dynamics decoupled from any mean-reverting fiscal rule, sustained only by high domestic savings and pre-EMU monetary arrangements. Finally, the 2020 episode identifies the "pandemic shock" as a systemic black swan event that temporarily pushed the debt-to-GDP ratio into explosive territory.

The core contribution of this research lies in the robust validation of these episodes through the $STADF$ and $GSTADF$ statistics. Our empirical analysis of the variance profile, reveals a stark non-linear accumulation of risk: the curve remains significantly below the homoskedastic benchmark for much of the early sample before exhibiting a near-vertical acceleration in the final stages. In such an environment—where approximately 80% of the total volatility is concentrated in the final 20% of the sample—standard recursive tests are prone to severe size distortions and spurious detections. However, our results demonstrate that the identified explosive regimes in Italy survive the time-transformation correction. This provides definitive evidence that these episodes are genuine structural deviations from the intertemporal budget constraint rather than statistical artifacts of late-sample heteroskedasticity.

Furthermore, while a formal time-transformed recursive sequence for real-time date-stamping remains an open area in econometric research, the high statistical significance of our global robust results serves as a critical filter for the standard BSADF identification. We conclude that accounting for the evolution of the variance profile is not merely a technical refinement, but an imperative requirement for the accurate monitoring of sovereign risk. Our findings offer a more nuanced and reliable identification of Italy's fiscal history, suggesting that while the debt has been historically resilient, the explosive nature of certain regimes underscores the persistent fragility of the Italian fiscal framework in the face of modern systemic turbulence.

Finally, our results provide critical insights for Italy's current fiscal outlook. While the 2020 explosive episode was temporary, the concentration of volatility in the recent segment of the sample suggests that the Italian fiscal framework remains highly sensitive to global shocks. Considering the European Commission's (2025b) projections, which foresee a renewed upward trend in the debt-to-GDP ratio for 2025 and 2026 due to the phase-out of temporary tax credits and rising debt-servicing costs, our findings act as a cautionary tale. The transition from a pandemic-driven 'stochastic' explosion to a potentially more 'structural' one—similar to the 1978–1998 regime—is a risk that policymakers must mitigate. In this sense, the implementation of robust monitoring tools like the $GSTADF$ is essential not only for historical analysis but also as an early-warning system. A credible commitment to the new European fiscal rules and the effective execution of the National Recovery and Resilience Plan (NRRP) will be decisive in preventing current debt dynamics from entering a new explosive phase in the post-pandemic high-interest-rate environment

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Table 1
Tests for explosive behavior: the $SADF$ and $GSADF$ tests [a,b]:
Italian public debt to GDP ratio, b_t

Unit root tests	Estimated Value	1%	5%	10%
$SADF$	0.950	2.416	1.669	1.351
$GSADF$	4.334*	1.328	0.603	0.244

Notes:
[a] Test the null of a unit root against the alternative of recurrent explosive behavior.
[b] *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively.

Table 2
Tests for non-normality and heteroskedasticity:
Italian public debt to GDP ratio, b_t

JB				LM_e			
Value	1%	5%	10%	Value	1%	5%	10%
33.43*	9.210	5.991	4.605	9.116*	6.634	3.841	2.705

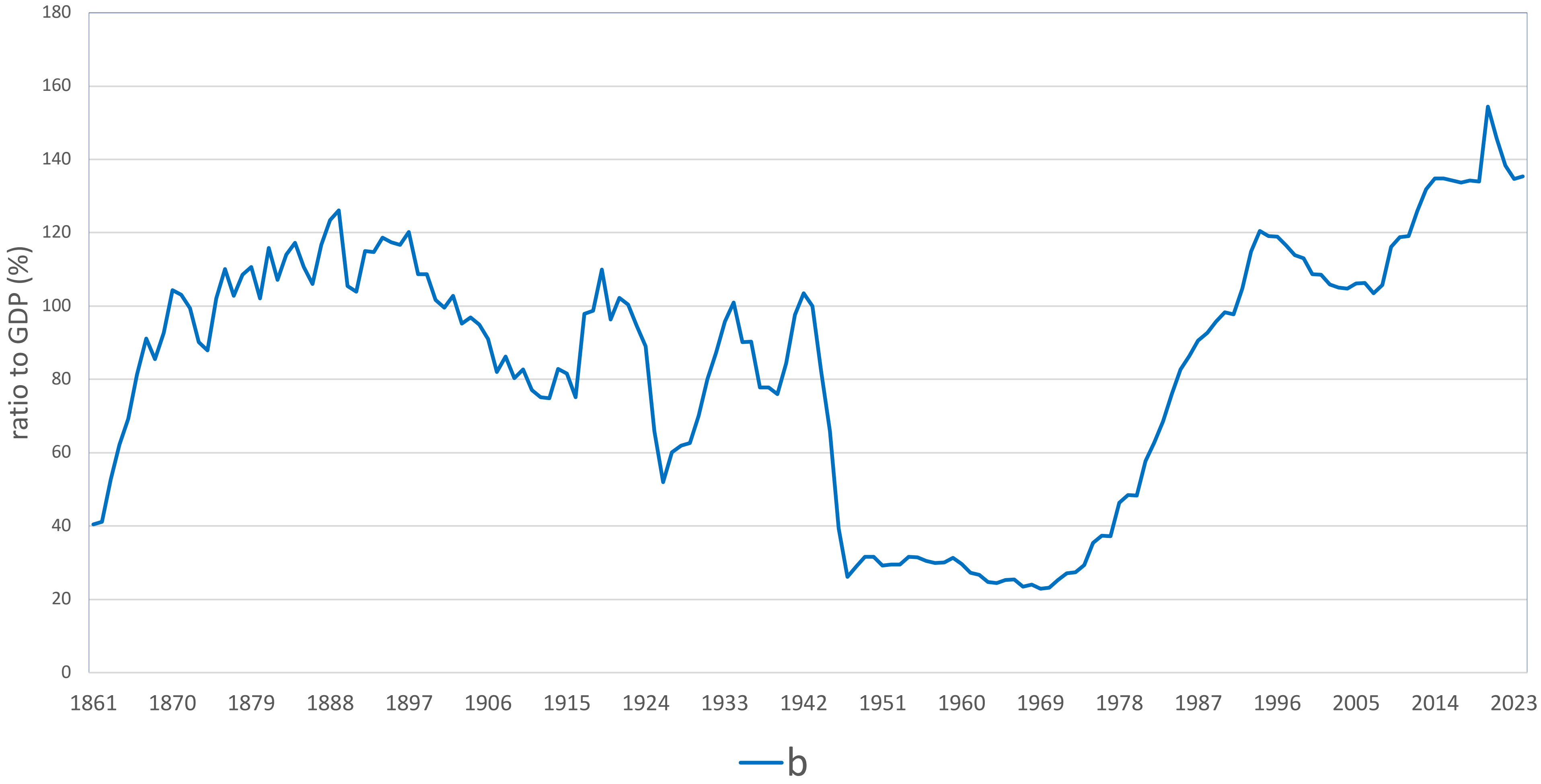
Notes: *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively.

Table 3
Test for explosive bubbles under non-stationarity volatility:
Italian public debt to GDP ratio, b_t,

Tests	p-values
$STADF$	0.007*
$GSTADF$	0.025**

Notes: p-values. *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively.

Figure 1. The Italian public debt-to-GDP
1861-2024



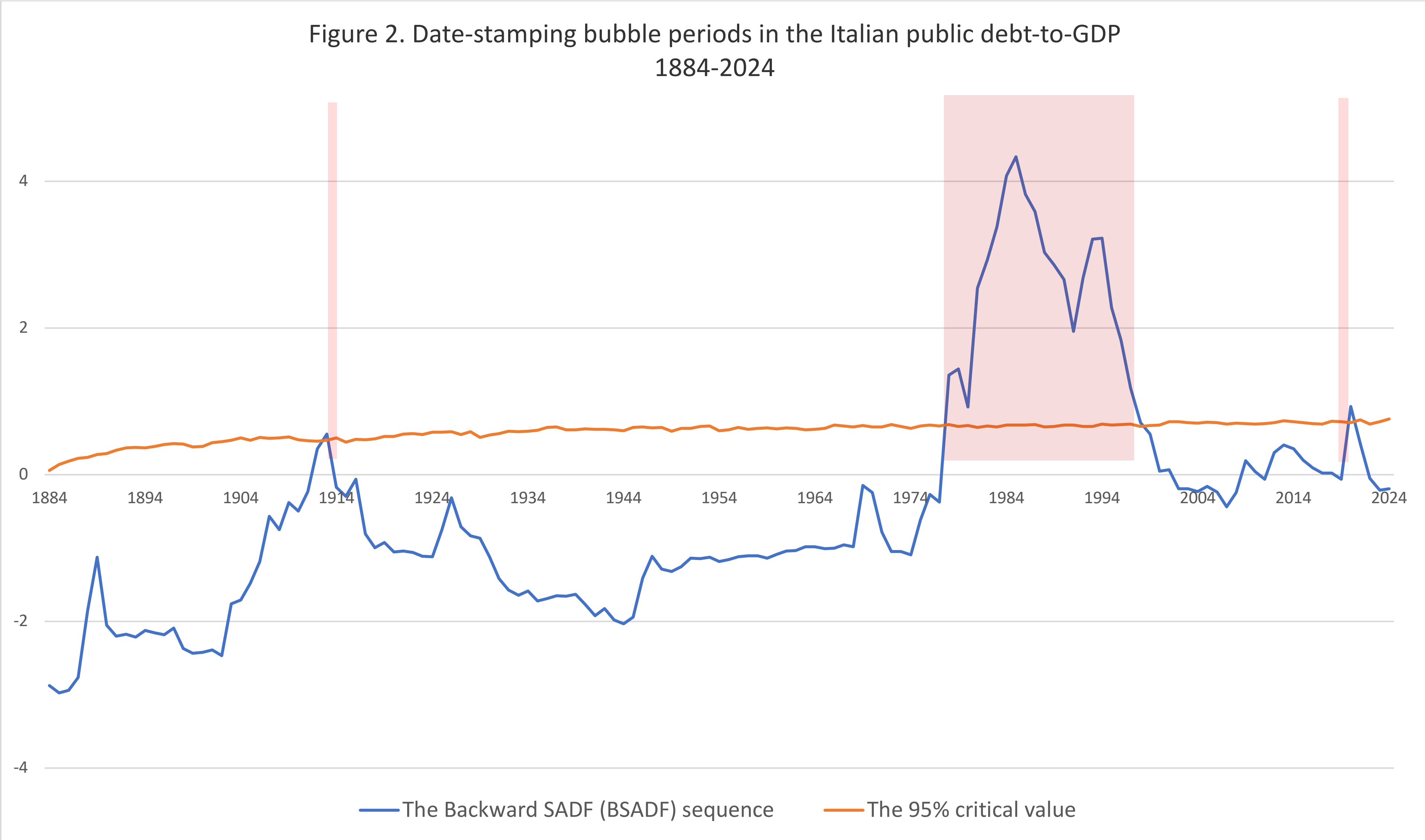


Figure 2. Date-stamping bubble periods in the Italian public debt-to-GDP
1884-2024
4
2
0
-2
-4
1884
1894
1904
1914
1924
1934
1944
1954
1964
1974
1984
1994
2004
2014
2024
The Backward SADF (BSADF) sequence
The 95% critical value

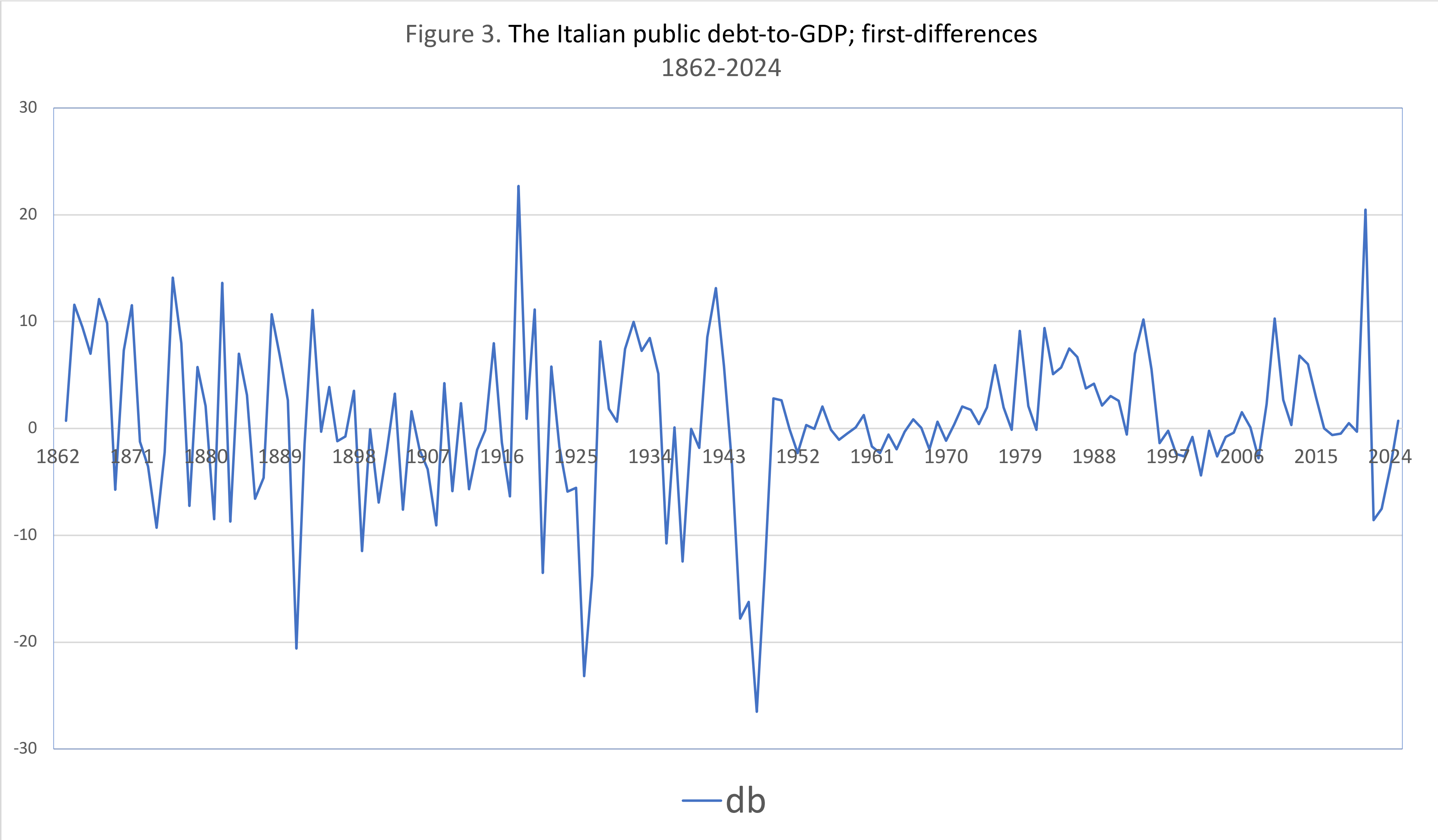


Figure 3. The Italian public debt-to-GDP; first-differences
1862-2024
30
20
10
0
-10
-20
-30
1862
1871
1880
1889
1898
1907
1916
1925
1934
1943
1952
1961
1970
1979
1988
1997
2006
2015
2024
db

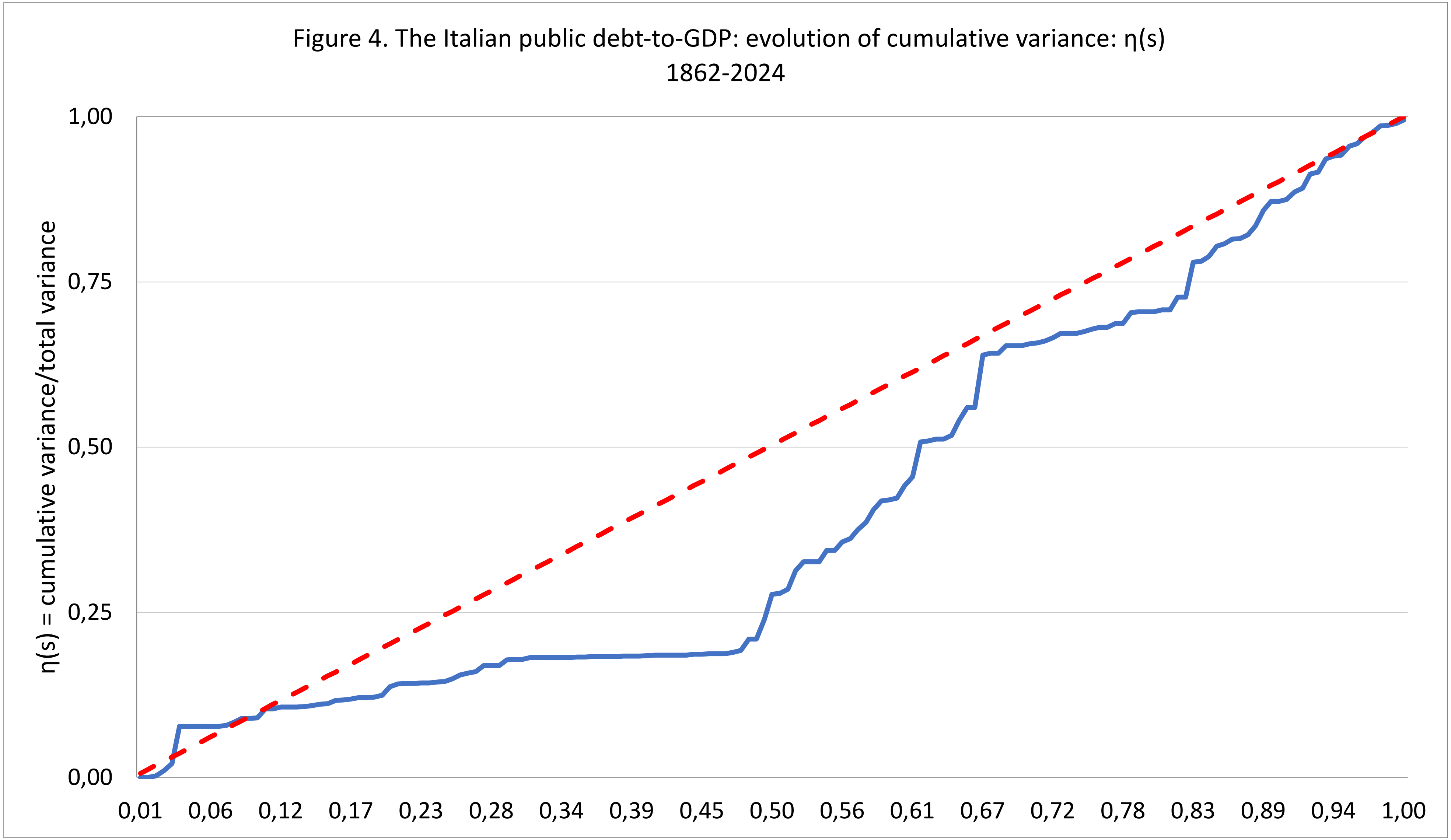


Figure 4. The Italian public debt-to-GDP: evolution of cumulative variance: η(s)
1862-2024
η(s) = cumulative variance/total variance
1,00
0,75
0,50
0,25
0,00
0,01
0,06
0,12
0,17
0,23
0,28
0,34
0,39
0,45
0,50
0,56
0,61
0,67
0,72
0,78
0,83
0,89
0,94
1,00